Comments on "Relaxed Conditions for the Input-to-State Stability of Switched Nonlinear Time-Varying Systems"

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***Abstract*—This study addresses the deficiencies in the assumptions of the results in Chen and Yang, 2017 [1] due to the lack of uniformity. We first show the missing hypothesis by presenting a counterexample. Then we prove why they are wrong in that form and show the errors in the proof of the main result of [1]. Next, we compare the assumptions and related results of [1] with similar works in the literature. Lastly, we give suggestions to complement the shortcomings of the hypotheses and thus correct them.**

***Index Terms*—Indefinite Lyapunov Function, Nonlinear Time-varying Systems, Uniform Asymptotic Stability, Uniform Attractivity.**

I. Introduction

We consider the nonlinear control system

$$\dot{x}(t) = f(t, x(t), u(t)), \quad x(t_0) = x_0, \quad t \geq t_0 \geq 0, \tag{1}$$

where $x(\cdot)$ is state of the system, $u(\cdot)$ is measurable and locally essentially bounded control input and $f : \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is locally Lipschitz in (t, x), uniformly continuous in u and satisfies $f(t, 0, 0) = 0$ for all $t \geq 0$.

The authors of [1] provide some relaxed hypotheses (compared to [10]) to guarantee input-to-state stability (ISS) and uniform asymptotic stability (UAS) for (1). The same results are then adapted to switched non-linear time-varying (SNTV) systems. However, the results have some missing assumptions that lead to imprecise conclusions. More precisely, uniformity, which is crucial for ISS and UAS, is not included in the hypotheses.

This work consists of six sections. Section 2 includes preliminaries. Section 3 explains with a counterexample why the hypotheses made in [1] do not imply UAS and therefore do not imply ISS. Section 4 shows what the errors are in the proof of the main results. Section 5 compares the conclusions of [1] with those similar to those in the literature and presents some recommendations to correct the errors. Section 6 gives the conclusions and the Authorship Contribution Statement.

Notations: We use the following abbreviations and definitions. $\mathbb{R}$ is the set of real numbers, $\mathbb{R}^+ := [0, \infty)$. $\mathcal{K}$, $\mathcal{K}_\infty$ and $\mathcal{KL}$ are the families of *class* $\mathcal{K}$, $\mathcal{K}_\infty$ and $\mathcal{KL}$ functions [4]. $|.|$ is the standard Euclidean norm. Some other abbreviations are also addressed when they are first mentioned.

II. Fundamental Definitions

We first give some preliminaries, [4], [14], and [17].

Definition 1. Consider the zero input case of (1). The equilibrium $x = 0$ is said to be:

- *Lyapunov stable* (or simply stable) if, for each $\epsilon > 0$ and $t_0 \geq 0$, there exists $\delta = \delta(\epsilon, t_0) > 0$ such that for all solutions $x(\cdot)$:

$$|x(t_0)| < \delta \implies \forall t \geq t_0 : |x(t)| < \epsilon; \tag{2}$$

- *uniformly stable (US)* if, for each $\epsilon > 0$ there is a $\delta = \delta(\epsilon) > 0$ such that for all $t_0 \geq 0$, all solutions $x(\cdot)$ and all $t \geq t_0$ the inequality (2) is satisfied, i.e. δ does not depend on t_0;
- *attractive* if for each $t_0 \geq 0$ there is a positive constant $c = c(t_0) > 0$ such that for all solutions $x(\cdot)$ with $|x(t_0)| < c$:

$$x(t) \to 0 \quad as \quad t \to \infty. \tag{3}$$

- *uniformly attractive (UA)* if there exists $c > 0$, independent of t_0, such that (3) holds uniformly for all $t_0 \geq 0$, that is, for each $\eta > 0$ there is a number $T = T(\eta) > 0$ such that for all $t_0 \geq 0$ and all solutions $x(\cdot)$:

$$|x(t_0)| < c \implies \forall t \geq t_0 + T(\eta) : |x(t)| < \eta. \tag{4}$$

- *asymptotically stable (AS)* if it is stable and attractive
- *uniformly asymptotically stable (UAS)* if it is US and UA

Definition 2. The equilibrium $x = 0$ of (1) is said to be *input-to-state stable (ISS)* if there exists $c > 0$ and $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}$ (all independent from t_0) such that for any $t_0 \geq 0$, any initial state $|x(t_0)| < c$, any bounded input $u(t)$, and any $t \geq t_0$, all solutions satisfy

$$|x(t)| \leq \beta(|x(t_0)|, t - t_0) + \gamma(\sup_{t_0 \leq s \leq t} |u(s)|). \tag{5}$$

It is clear that if a system is ISS, it must already be UAS when $u = 0$. However, the converse is not true, in general. There exist UAS systems that do not satisfy the ISS property (see Section 4.9 of [4] and 1.3.3 of [7]). Some more equivalent conditions can be given via comparison functions [4].

Lemma 1. *Consider the zero input case of (1). The equilibrium point $x = 0$ is UAS if and only if there exist $\beta \in \mathcal{KL}$ and $c > 0$, both independent of t_0, such that for all $t_0 \geq 0$, all $t \geq t_0$ and all solutions $x(\cdot)$ with $|x(t_0)| < c$:*

$$|x(t)| \leq \beta(|x(t_0)|, t - t_0). \tag{6}$$

Remark 1 (Uniformity). Now we explain the uniformity concept in more detail. Nonlinear time-varying (NTV) (or nonautonomous) systems have a distinguishing feature compared to nonlinear time-invariant (NTI) (or autonomous) systems: *"uniformity"*. While the solution of an NTI system depends only on $t - t_0$ by default, it depends on both t and t_0 for NTV systems (see p.148 of [4]). Uniformity is related to the independence of t_0 in the definitions of stability (2) and attractivity (3). This property may be useful for NTV systems when looking at stability and attractivity properties. On the other hand, if we do not utilize a strict Lyapunov function, it is challenging to check the uniformity of a time-varying system, [12]. For attractivity, i) the radius of the ball of attraction and ii) the convergence to the equilibria generally depends on t_0. Different variants of these conditions are studied in the literature in detail, [5], [8], and [9]. Consequently, providing **UA**, the initial time t_0 affects neither the asymptotic convergence rate of solutions of (1) to the equilibrium nor the radius of the ball of attraction; see [2, p. 828].

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III. Mistakes in [1] and counterexample

We first present the original statements from [1].

Assertion 1 (Theorem 2 of [1])**.** *Consider the system (1). Suppose there exist a continuously differentiable function $V : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^+$, two class $\mathcal{K}_\infty$ functions α_1 and α_2, a class $\mathcal{K}$ function ρ, a continuous function $g : \mathbb{R}^+ \to \mathbb{R}$, and a constant $M \geq 0$ such that*

1) $\alpha_1(|x|) \leq V(t,x) \leq \alpha_2(|x|)$,
2) $\dot{V}(t,x) \leq g(t)V(t,x)$ *for* $V(t,x) \geq \rho(|u(t)|)$,
3) $\int_{t_0}^{\infty} g(t)dt = -\infty$,
4) $\int_s^t g(\tau)d\tau \leq M, \forall t \geq s \geq t_0$.

Then the system (1) is ISS with $\gamma(s) = \alpha_1^{-1}(2e^M \rho(s))$.

As a corollary of this result, the following conclusion is given in [1].

Assertion 2 (Corollary 1 of [1])**.** *Consider the system (1) with zero input. Suppose there exist a continuously differentiable function $V : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^+$, two class $\mathcal{K}_\infty$ functions α_1 and α_2, and a continuous function $g : \mathbb{R}^+ \to \mathbb{R}$ such that*

1) $\alpha_1(|x|) \leq V(t,x) \leq \alpha_2(|x|)$,
2) $\dot{V}(t,x) \leq g(t)V(t,x)$ *with* $\int_{t_0}^{\infty} g(t)dt = -\infty$.

Then the system (1) is UAS.

We first note that in both assertions (replicated unaltered from the source) the quantors for the variables x, t, t_0 are missing; our interpretation is that in all cases "for all x / t / t_0" is added. Assertion 1 claims that the conditions from 1) to 4) imply ISS and so UAS for $u = 0$. Therefore, these conditions must satisfy that $x = 0$ is both US and UA. However, we will clearly show that they only provide US and attractivity, not UA. More precisely, the conditions of Assertion 1 do not satisfy uniformity in attractivity. Hence, they cannot guarantee ISS. The same problem is valid also for Assertion 2.

We now express the main problem in [1] mathematically but leave the detailed error analysis of the proof to Section 4. In the proof of Assertion 1 of [1], for $u = 0$, the following inequality can be received.

$$V(t,x(t)) \leq V(t_0,x(t_0))e^{\int_{t_0}^t g(s)ds}. \tag{7}$$

This leads us to the following estimate by item 1 of Assertion 1:

$$|x(t)| \leq \alpha_1^{-1}\big[\alpha_2\big(|x(t_0)|\big)e^{\int_{t_0}^t g(s)ds}\big], \tag{8}$$

where $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$. This is the zero input version of (13) of [1]. We know by Lemma 1 above that to receive UAS, it is necessary and sufficient that the solution must be bounded by a $\beta(|x(t_0)|, t-t_0) \in \mathcal{KL}$. There shouldn't exist any other t or t_0 term in β except the ones in $t - t_0$. The right-hand side of (8) is tried to make a $\mathcal{KL}$ function in [1]. By defining $G(t) := \int_{t_0}^t g(s)ds$, we have

$$\alpha_1^{-1}\big[\alpha_2\big(|x(t_0)|\big)e^{G(t)}\big] \leq \alpha_1^{-1}\big[\alpha_2\big(|x(t_0)|\big)e^{M}\big] \in \mathcal{K}.$$

So the UA of the equilibrium is provided by the available conditions of Assertion 1, (see Lemma 4.5 of [4]). But item 3 of Assertion 1 (this is the condition $G(t) \to -\infty$ as $t \to \infty$) just implies that the right-hand side of (8) is a *class* $\mathcal{L}$ function of t, and generally it is not a function of $t - t_0$. Thus, G or another upper bound of it should be limited by an $\alpha(t - t_0) \in \mathcal{L}$. Although this necessity was noticed in the proof of Assertion 1, it was forgotten in the conditions of Assertions 1 and 2 and the rest of the proof is not true. Thus, both item 3 of Assertion 1 and item 2 of Assertion 2 may fail to imply UA. They provide just attractivity. We reinforce our claim with a known example; see [4, Ex. 4.18] and [7, Ex. 1.3.5].

Example 1: Consider the following equation

$$\dot{x} = -\frac{1}{1+t}x. \tag{9}$$

Choose $V(x) = x^2$. Then

$$\dot{V} = 2x\dot{x} = -\frac{2}{1+t}V \leq g(t)V$$

with $g(t) = -\frac{1}{1+t}$. Choosing $\alpha_1(|x|) = \frac{1}{2}x^2$, $\alpha_2(|x|) = 2x^2$, all the conditions of Assertion 1 and 2 hold. However, this system is known to be not UAS. In fact, the solution is explicitly given by

$$x(t) = x(t_0)\frac{1+t_0}{1+t}. \tag{10}$$

It is claimed in [4] and [7] that this system is not UAS, but in order to be self-contained we provide the mathematical argument for that claim here as well. Note that the example is US (just choose $\delta := \epsilon$) and attractive, but attractivity is not uniform, because (4) cannot be satisfied. In fact, for the solution (10) to satisfy (4) for all initial values $|x(t_0)| < c$ we need to choose $T(\eta) > (c - \eta)(1 + t_0)/\eta$ which, for $\eta < c$, clearly grows unbounded for $t_0 \to \infty$; thus the convergence is not uniform. Some interesting counterexamples can be found in the literature for the following variants, as well:

- A system may be "UA but not US", see [5, Ex. 5],
- Attractivity even may not imply stability, see [13, Sec. 3.6],

(see also the references [3], [7], [11, Ex. 6.11], [4, Sec. 4.5], [8, Exs. 1-2], and [15, Exs. 1-5]). This shows that NTV systems have many variants from the point of view of uniformity.

Now, let us review Assertion 2. We have the following observations for this result.

- Attractivity is given in the same way by an improper integral condition without implying the uniformity property.
- Moreover, the boundedness condition for the integral of $g(t)$, i.e. item 4 of Assertion 1, has been completely removed. This could prevent the system (1) from being US, see [13, Sec. 3.6] as already emphasized above.

Remark 2 (Comments on the rest of the results of [1])*.* The same problems continue for the following conclusions of Section IV of [1]. In that section, the results of the previous section are tried to adapt to SNTV systems. $G(t)$ is improved such that it includes also the number of the switches $\lambda(t, t_0)$. In that case,

$$\overline{G}(t) = \lambda(t,t_0)\ln b + \int_{t_0}^t g_{\sigma(t)}(\tau)d\tau \tag{11}$$

is required to converge to $-\infty$ [1, Theorem 3]. But that convergency again is not uniform. The following theorems and corollaries of the same section can be regarded as problematic in the same fashion.

IV. The Error in the Proof of the Main Result

In this section we would like to highlight the error in the proof of Assertion 1 given in [1]. Therein the authors consider the antiderivative of g, denoted $G(t) := \int_{t_0}^t g(\tau)\,\mathrm{d}\tau$, and a first problem is the omittance of the dependence on the initial time t_0. To highlight this dependence, we use in the following the notation $G(t, t_0)$ instead; for the above counter-example, we have $G(t, t_0) = \ln\frac{1+t_0}{1+t}$. They then fix an upper bound $c \geq 0$ for G which in fact can be chosen independently from t_0 as $c = M$, where M is a uniform bound for $G(t, s)$ assumed to exist in item 4 of Assertion 1 (in fact, item 2 of Assertion 1 can also be utilized to bound G, however, the independence from t_0 is then not so clear). As a next step the authors of [1] postulate the existence of a sequence $\{t_k\}$ such that t_k converges to infinity for $k \to \infty$ and

$$G(t,t_0) \leq c - kh \quad \text{for } t \geq t_k$$

for some $h > 0$. At this point, it is unclear whether the authors allow h to be dependent on t_0 or not. In any case, already at this point, there is a fundamental problem with notation, because clearly the authors assume that $t_k \geq t_0$ for all k, but this means that the sequence actually needs to depend on t_0! Furthermore, the authors continue to define a function $H(t-t_0) = c-(k-1)h - \frac{t-t_k}{t_{k+1}-t_k}h$ for $t-t_0 \in [t_k - t_0, t_{k+1}-t_0)$, which clearly is not just a function of the argument $t-t_0$ bu but also depends on t_0 because $t-t_k = (t-t_0)-(t_k-t_0)$ and also the boundaries of the piecewisely defined function H depend on t_0. However, this oversight can be fixed easily by instead considering a relative time-sequence $\{s_k\}$ replacing the absolute time sequence $\{t_k\}$ such that $t_k = t_0 + s_k$. The construction of H takes then the form

$$H(t-t_0) = c-(k-1)h - \frac{t-t_0-s_k}{s_{k+1}-s_k}h, \quad \text{for } t-t_0 \in [s_k, s_{k+1}).$$

Together with the adjusted assumption on $h > 0$ and $c \geq 0$ given by

$$G(t,t_0) \leq c - kh \quad \text{for } t \geq t_0 + s_k \tag{12}$$

we can indeed conclude that

$$G(t,t_0) \leq H(t-t_0).$$

Based on this inequality the authors of [1] then construct a $\mathcal{KL}$-function bounding the state x with a convergence to zero only depending on $t-t_0$, which would indeed show uniform convergence. However, to arrive at this conclusion it is important that $h > 0$ as well as the actual time sequence $\{s_k\}$ can be chosen independently from t_0 (so that H is indeed only a function of $t-t_0$). *But this exactly is the error of the proof, because such a choice is not always possible!* This can be seen by considering the above-given counterexample: Clearly, we can choose $c = 0$ and if we assume that there is a $h > 0$ and a sequence $\{s_k\}$ such that (12) holds, then, in particular for $k = 1$,

$$G(s_1 + t_0, t_0) \leq -h < 0.$$

However, we see that $G(s_1 + t_0, t_0) = \ln \frac{1+t_0}{1+s_1+t_0} \to 0$ as $t_0 \to \infty$, i.e. for sufficiently large t_0 we have that $G(s_1 + t_0, t_0) > -h$. Consequently, it is *not possible* to choose $h > 0$ and a sequence $\{s_k\}$ *independently* of t_0 such that (12) holds.

V. Literature Review and fixing the results from [1]

To fix the Assertions of [1],

1) all items of Assertion 1 and 2 should include the necessary quantifiers such as "for all x", "for all t" and/or "for all t_0";
2) the uniformity property should be incorporated into item 3 of Assertion 1 and item 2 of Assertion 2;
3) item 4 of Assertion 1 also needs to be included in Assertion 2.

The second fix can be realized in different ways.

1) For example, the work [6] that cites [1], uses an attractivity condition similar to item 3 of Assertion 1. The work [6] studied the stability of SNTV systems. For US, [6] uses boundedness:

$$\int_{t_0}^{t} \left(\frac{\ln(\gamma)}{\tau} + g(s) \right) ds < \lambda \in \mathbb{R}^+ \tag{13}$$

in Theorem 3 of [6] where τ is the average dwell time, $V_i \leq \gamma V_j$. When uniformity is needed for attractivity, i.e. for UA, Theorem 4 of [6] gives a similar condition with item 3 of Assertion 1 but [6] especially emphasizes uniformity:

$$\lim_{t\to\infty} \int_{t_0}^{t} \left(\frac{\ln(\gamma)}{\tau} + g(s) \right) ds = -\infty, \quad \textit{uniformly in } t_0. \tag{14}$$

But Assertion 1 item 3 does not include uniformity.
Even the expressions *"uniformly in t_0"* and *"at a uniform rate"* are used to help to identify uniform convergence by many authors quite commonly, p.150 of [4], p.140 of [14], they can be regarded too imprecise. Thus, we also serve a definition-based statement. The missing uniform convergence part in Assertions 1 and 2 can also be fixed as follows:
If for all $\eta > 0$ there exists a number $T = T(\eta) > 0$ such that for all $t_0 \geq 0$, all solutions $x(\cdot)$ with $|x(t_0)| < c$ and all $t \geq t_0 + T(\eta)$:

$$\int_{t_0}^{t} g(\lambda)d\lambda < \ln \frac{\alpha_1(\eta)}{c}. \tag{15}$$

2) Now, let us consider the work [16] that considers almost the same problems. We first give some preliminaries on notations, [15] and [16]. If the equilibrium $y = 0$ of a linear time-varying (LTV) system,

$$\dot{y}(t) = \mu(t)y(t), \quad t \in [0,\infty). \tag{16}$$

is US (or UA), then μ is called a *US (or UA) function*. Now, we continue with Theorems 1 and 3 of [16]. Here, the same system (1) is considered, and items 1 and 2 of Assertion 1 are assumed. In addition, the coefficient g in Assertions 1 and 2 (which is named μ in [16]) is assumed to be a UAS function (or equivalently Uniformly Exponentially Stable-UES, [11]) in Theorems 1 and 3 of [16]. This means that the equilibrium $y = 0$ of the system (16) is US and UA. Then [16] defines

$$\alpha_1^{-1}\left[\alpha_2\big(|x(t_0)|\big)e^{\int_{t_0}^{t}\mu(\lambda)d\lambda}\right] := \beta(|x(t_0)|, t-t_0) \in \mathcal{KL}$$

(this follows from the fact that μ is a UAS function). Therefore, the rest of the conditions in Assertion 1 and 2, that is, the conditions 3 and 4 of Assertion 1 must have been equivalent to UAS of $g(t)$. That is, US and UA of $g(t)$ are represented by items 4 and 3 of Assertion 1, respectively.

In summary, the missing uniformity assumption in Assertion 1 can be fixed using one of the following options.

- Adding the expression "uniformly in t_0" to item 3 as in the condition (14), but preferably with an additional clarification of what "uniformly in t_0" precisely means, e.g. condition (15).
- Adding the condition that $g(t)$ is a UAS function instead of conditions 3 and 4.

Remark 3 (Comments on Examples of [1]). The work [1] also includes an example in Section V to show the efficiency of the results. The example is chosen as an LTV control system and the coefficient matrix $A(t)$ is chosen to be periodic. AS, ES and uniform exponential stability are all equivalent concepts for periodic LTV systems [15, Lemma 5]. Therefore, it is quite expected that the example verifies the claims because uniformity is implied by the periodicity.

VI. Conclusion

This article addresses some mistakes concerning the work [1]. We emphasize the importance of uniformity in the analysis of NTV Systems. This study also suggests some corrections to the provided conditions and proofs.

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References

[1] Chen G. and Yang Y. "Relaxed Conditions for the Input-to-State Stability of Switched Nonlinear Time-Varying Systems". *IEEE Transactions of the Automatic Control*. vol.62(9), pp. 4706-4712, 2017.

[2] Jitsuro S., Kazuya K. "Uniform global asymptotic stability for oscillators with superlinear damping". *Journal of Mathematical Analysis and Applications*. Volume 425, Issue 2, pp. 827-853, 2015.

[3] Sugie, Jitsuro. "Uniform global asymptotic stability for nonautonomous nonlinear dynamical systems". *Journal of Mathematical Analysis and Applications*. 519.1, Article No:126768, 2023.

[4] Khalil Hassan. *Nonlinear systems*. Macmillan Publishing Company, 1992.

[5] Lewis, Andrew. "Remarks on Stability of Linear Time-Varying Systems". *IEEE Transactions on Automatic Control*. Vol.62(11), pp.6039-6043, 2017.

[6] Lu, J, She, Z. "Average dwell time based stability analysis for nonautonomous continuous-time switched systems". *Int J Robust Nonlinear Control*.Vol:29, pp. 2333–2350, 2019.

[7] Malisoff, M. and Mazenc, F. *Constructions of Strict Lyapunov Functions*. Springer Science & Business Media, 2009.

[8] Maggiore, M., Antonio L., and Elena P. "Reduction theorems for stability of compact sets in time-varying systems". *Automatica* 148, Article No: 110771, 2023.

[9] Mironchenko, Andrii. "Uniform weak attractivity and criteria for practical global asymptotic stability". *Systems & Control Letters* 105: pp.92-99, 2017.

[10] Ning C., Yong He, Wu Min, Liu Q., She J., "Input-to-state stability of nonlinear systems based on an indefinite Lyapunov function". *Systems and Control Letters*, vol.61, pp.1254-1259, 2012.

[11] Rugh, Wilson J. *Linear System Theory (2nd ed.)*. Upper Saddle River, New Jersey: Prentice Hall, 1996.

[12] T. -C. Lee, Y. Tan, and I. Mareels, "Limiting Behavior of Hybrid Time-Varying Systems," in IEEE Transactions on Automatic Control, vol. 67, no. 11, pp. 5777-5792, Nov. 2022, doi: 10.1109/TAC.2021.3122376.

[13] Terrell, William J. *Stability and Stabilization, An Introduction*. Princeton University Press, New Jersey, USA, 2009.

[14] Vidyasagar M. *Nonlinear Systems Analysis.* SIAM, PA, USA, 2002.

[15] Zhou, B., "On asymptotic stability of linear time-varying systems". *Automatica*, vol.68, pp.266-276, 2016.

[16] Zhou, B., "Stability analysis of non-linear time-varying systems by Lyapunov functions with indefinite derivatives". *IET Control Theory and App.*, vol.11(9), pp.1434-1442, 2017.

[17] Z.M. Wang, Y. Tan, G.X. Wang, D. Nešić. "On Stability Properties of Nonlinear Time-Varying Systems by Semi-definite Time-Varying Lyapunov Candidates". *IFAC Proceedings Volumes*. Vol.41(2), pp.1123-1128, 2008.