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Linear quadratic optimal control of switched differential algebraic equations

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Abstract: In this abstract the finite horizon linear quadratic optimal control problem with constraints on the terminal state for switched differential algebraic equations is considered. Furthermore, we seek for an optimal solution that is impulse-free. In order to solve the problem, a non standard finite horizon problem for non-switched DAEs is considered. Necessary and sufficient conditions on the initial value x_0 for solvability of this non standard problem are stated. Based on these results a sequence of subspaces can be defined which lead to necessary and sufficient conditions for solvability of the finite horizon optimal control problem for switched DAEs.

Keywords: Switched systems, Differential Algebraic Equations, Optimal control, Linear systems.

1. INTRODUCTION

In this abstract we consider the following switched differential algebraic system

$$\begin{aligned} E_\sigma \dot{x} &= A_\sigma x + B_\sigma u \\ y &= C_\sigma x + D_\sigma u. \end{aligned} \tag{1}$$

where $\sigma : \mathbb{R} \to \mathbb{N}$ is the switching signal and $E_p, A_p \in \mathbb{R}^{n\times n}$ and (E_p, A_p) is regular, $B_p \in \mathbb{R}^{n\times m}$, $C_p \in \mathbb{R}^{q\times n}$ and $D_p \in \mathbb{R}^{p\times m}$ for $p, q, n, m \in \mathbb{N}$. We aim to find an impulse-free solution (x, u) on $[t_0, t_f)$ satisfying $x(t_0^-) = x_0$ and $x(t_f^-) \in \mathcal{V}^{\text{end}}$ that minimizes

$$J(x_0, u, t_f) = \int_{t_0}^{t_f} y(t)^\top y(t)\ \mathrm{d}t + x(t_f^-)^\top P x(t_f^-) \tag{2}$$

for some positive semi definite $P = P^\top \in \mathbb{R}^{n\times n}$ and y is the output resulting from the solution (x, u) of (1) satisfying $x(t_0^-) = x_0$. In general, trajectories of switched DAEs exhibit jumps (or even impulses), which may exclude classical solutions from existence. Therefore, we adopt the *piecewise-smooth distributional solution framework* introduced in Trenn (2009).

Differential algebraic equations (DAEs) arise naturally when modeling physical systems with certain algebraic constraints on the state variables. Examples of applications of DAEs in electrical circuits can be found e.g. in Tolsa and Salichs (1993); Riaza (2008); Reis (2010) and gas networks, where the algebraic constraints are induced by the network topology. *e.g.* in Grundel et al. (2014). The algebraic constraints are often eliminated such that the system is described by ordinary differential equations (ODEs). However, in the case of switched systems, the elimination process of the constraints is in general different for each individual mode. Therefore, in general, there does not exist a description as a switched ODE with a common state variable for every mode. This problem can be overcome by studying switched DAEs directly.

The literature on optimal control of non-switched DAEs is quite mature, (besides the already mentioned literature) see for the finite horizon e.g. Kunkel and Mehrmann (2008, 1997); Ilchmann et al. (2019, 2021); Wijnbergen and Trenn (2021b) on a finite horizon and for the infinite time horizon see e.g. Cobb (1983); Mehrmann (1989); Reis et al. (2015); Reis and Voigt (2019); Bankmann and Voigt (2019). Furthermore, several structural properties of switched DAEs have been investigated recently (Wijnbergen and Trenn, 2021a, 2020). However, to the best of the authors knowledge, optimal control of switched DAEs has not been studied yet.

The finite horizon problem is motivated by the study of optimal control on an infinite horizon, *i.e.,* the minimization of

$$J(x_0, u) = \int_{t_0}^{\infty} y(t)^\top y(t)\ \mathrm{d}t. \tag{3}$$

on the interval $[t_0, \infty)$ while using a dynamic programming approach. It can be shown that if there exists an input that minimizes (3), the optimal cost resulting from the interval $[t_f, \infty)$ is quadratic in $x(t_f^-)$, *i.e.,* $x(t_f^-)^\top P x(t_f^-)$ for some matrix $P \in \mathbb{R}^{n\times n}$. This result allows for a dynamic programming approach. Assuming that the matrix P and the optimal input u restricted to $[t_f, \infty)$ are known, it follows that the minimization of (3) is equivalent to finding the input u restricted to $[t_0, t_f)$ such that

$$\begin{aligned} J(x_0, u) &= \int_{t_0}^{t_f} y(t)^\top y(t)\ \mathrm{d}t + \int_{t_f}^{\infty} y(t)^\top y(t)\ \mathrm{d}t \\ &= \int_{t_0}^{t_f} y(t)^\top y(t)\ \mathrm{d}t + x(t_f^-)^\top P x(t_f^-). \end{aligned}$$

is minimal.

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For many real word applications Dirac impulses in the state are to be avoided as they can cause damage to components of the system or create hazardous situations. Therefore, we aim to find an optimal *impulse-free* solution (x, u). However, there generally only exists an optimal (impulse-free) solution on $[t_f, \infty)$ if the state at t_f is contained in some subspace, *i.e.,* $x(t_f^-) \in \mathcal{V}^{\text{end}}$ for some subspace $\mathcal{V}^{\text{end}} \subseteq \mathbb{R}^n$.

In order to solve the problem for switched systems with a switching signal that induces an arbitrary yet finitely many modes on the interval $[t_0, t_f)$ we will first consider the case that only two modes are induced on $[t_0, \infty)$ and the switch occurs at t_f. Within this context we aim to minimize (2) with respect to a non-switched DAE. Once conditions for the single switched case are obtained, conditions for the general case will follow straightforwardly. Hence first we will focus on finding an optimal solution to

$$E\dot{x} = Ax + Bu, \tag{4}$$

$$y = Cx + Du \tag{5}$$

that minimizes (2) under the constraint $x_0 \in \mathbb{R}^n$ and $x(t_f^-) \in \mathcal{V}^{\text{end}}$.

As the terminal cost matrix P represents the cost resulting from the interval $[t_f, \infty)$ and the mode active on this interval is not necessarily structurally related to the dynamics (4), we can only assume that $P \in \mathbb{R}^{n\times n}$ is some positive semi-definite matrix. This is in contrast to the assumption commonly made in the literature for optimal control of non switched DAEs that the terminal cost matrix is of the form $P = E^\top \tilde{P} E$ for some positive semi-definite $\tilde{P} \in \mathbb{R}^{n\times n}$ (Lewis, 1985; Bender and Laub, 1985; Katayama and Minamino, 1992). Also note that whereas commonly a closed interval is of interest, in this paper a half open interval is considered. Consequently, the terminal cost can penalizes algebraic states and as a result $x(t_f^-)$ is not necessarily equal to $x(t_f)$ or even well defined such that an optimal solution might fail to exist.

The remainder of this paper is structured as follows. The mathematical preliminaries and the main results are given in Section 2 and 3, respectively. Conclusions and a discussion on future work are given in Section 4.

2. MATHEMATICAL PRELIMINARIES

In the following, we consider *regular* matrix pairs (E, A), i.e. for which the polynomial $\det(sE - A)$ is not the zero polynomial. Recall the following result on the *quasi-Weierstrass form (QWF)* (Berger et al., 2012).

Proposition 1. A matrix pair $(E, A) \in \mathbb{R}^{n\times n} \times \mathbb{R}^{n\times n}$ is regular if, and only if, there exists invertible matrices $S, T \in \mathbb{R}^{n\times n}$ such that

$$(SET, SAT) = \left(\begin{bmatrix} I & 0 \\ 0 & N \end{bmatrix}, \begin{bmatrix} J & 0 \\ 0 & I \end{bmatrix} \right), \tag{6}$$

where $J \in \mathbb{R}^{n_1\times n_1}$, $0 \leqslant n_1 \leqslant n$, is some matrix and $N \in \mathbb{R}^{n_2\times n_2}$, $n_2 := n - n_1$, is a nilpotent matrix.

The matrices S and T can be calculated by using the so-called *Wong sequences* (Berger et al., 2012; Wong, 1974): Based on the Wong sequences we define the following projectors and selectors.

Definition 2. Consider the regular matrix pair (E, A) with corresponding quasi-Weierstrass form (6). The *consistency projector* of (E, A) is given by

$$\Pi := T \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} T^{-1},$$

the *differential selector* and the *impulse selector* are given by

$$\Pi^{\text{diff}} := T \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} S, \quad \Pi^{\text{imp}} := T \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix} S.$$

respectively

In all three cases the block structure corresponds to the block structure of the QWF. Furthermore we define

$$A^{\text{diff}} := \Pi^{\text{diff}} A, \qquad E^{\text{imp}} := \Pi^{\text{imp}} E,$$
$$B^{\text{diff}} := \Pi^{\text{diff}} B, \qquad B^{\text{imp}} := \Pi^{\text{imp}} B.$$

Note that all the above defined matrices do not depend on the specifically chosen transformation matrices S and T; they are uniquely determined by the original regular matrix pair (E, A). An important feature for DAEs is the so called consistency space, defined as follows for the DAE given in (4).

Definition 3. Consider the DAE (4), then the *consistency space* is defined as

$$\mathcal{V}_{(E,A)} := \left\{ x_0 \in \mathbb{R}^n \;\middle|\; \begin{array}{l} \exists \text{ smooth solution } x \text{ of (4)} \\ \text{with } u = 0 \text{ and } x(0) = x_0 \end{array} \right\},$$

and the *augmented consistency space* is defined as

$$\mathcal{V}_{(E,A,B)} := \left\{ x_0 \in \mathbb{R}^n \;\middle|\; \begin{array}{l} \exists \text{ smooth solutions } (x, u) \text{ of (4)} \\ \text{with } x(0) = x_0 \end{array} \right\}.$$

For studying impulsive solutions of (4), we consider the space of *piecewise-smooth distributions* $\mathbb{D}_{\text{pw}\mathcal{C}^\infty}$ from Trenn (2009) as the solution space. That is, we seek a solution $(x, u) \in (\mathbb{D}_{\text{pw}\mathcal{C}^\infty})^{n+m}$ to the following initial-trajectory problem (ITP) associated with (4):

$$x_{(-\infty,0)} = x^0_{(-\infty,0)}, \tag{7a}$$

$$(E\dot{x})_{[0,\infty)} = (Ax)_{[0,\infty)} + (Bu)_{[0,\infty)}, \tag{7b}$$

where $x^0 \in (\mathbb{D}_{\text{pw}\mathcal{C}^\infty})^n$ is some initial trajectory, and $f_{\mathcal{I}}$ denotes the restriction of a piecewise-smooth distribution f to an interval $\mathcal{I}$. In Trenn (2009) it is shown that the ITP (7) has a unique solution for any initial trajectory if, and only if, the matrix pair (E, A) is regular. As a direct consequence, the switched DAE considered in (1) with regular matrix pairs is also uniquely solvable (with piecewise-smooth distributional solutions) for any switching signal with locally finitely many switches.

The space of initial values for which there exists an impulse-free solution (x, u) of (4) is defined as follows.

Definition 4. Consider the DAE (4), then the *impulse controllable space* is defined as

$$\mathcal{C}^{\text{imp}} := \left\{ x_0 \in \mathbb{R}^n \;\middle|\; \begin{array}{l} \exists \text{ solution of (7) satisfying} \\ x(0^-) = x_0 \text{ and } (x, u)[0] = 0 \end{array} \right\},$$

The DAE is called impulse controllable if $\mathcal{C}^{\text{imp}} = \mathbb{R}^n$.

Lemma 5. (Cobb (1981)). The DAE (4) is impulse controllable if and only if there exist a feedback input $u = Lx + v$ such that in terms of the selectors Π^{diff} and Π^{imp} resulting from the Wong sequences based on $(E, A + BL)$, the solution x is given by $x = x^{\text{diff}} - B^{\text{imp}} v$ where x^{diff} solves

$$\dot{x}^{\text{diff}} = (A + BL)^{\text{diff}} x^{\text{diff}} + B^{\text{diff}} v \tag{8}$$

Given a matrix pair (E, A) and a matrix B we can always generate an impulse-controllable DAE with the same solution behavior for initial values $x_0 \in \mathcal{C}^{\text{imp}}$.

Lemma 6. Consider the DAE (4). A solution (x, u) satisfying $x(0^-) = x_0 \in \mathcal{C}^{\text{imp}}$ solves (4) if and only if (x, u) solves

$$EW\dot{x} = Ax + Bu \tag{9}$$

where W is an orthogonal projector onto $\mathcal{C}^{\text{imp}}$. In addition, (9) is impulse controllable.

3. MAIN RESULTS

The concepts introduced in the previous section are now utilized to study the minimization of (2) subject to the non-switched DAE (4). We aim to minimize (2) over all (x, u) that are impulse-free and satisfy $x(t_0^-) = x_0$ and $x(t_f^-) \in \mathcal{V}^{\text{end}}$.

As we aim to find an impulse-free solution (x, u) that minimizes (2), it is necessary that $x(t_0) = x_0 \in \mathcal{C}^{\text{imp}}$. Consequently, it follows from Lemma 6 that we can assume without loss of generality that (4) is impulse controllable. Moreover, we can assume that an index-reducing feedback in the sense of Lemma 5 has been applied and that the system is of index-1.

In the case (4) is of index-1, we observe, by making use of the decomposition $x = x^{\text{diff}} - B^{\text{imp}}u$, that (x, u, y) solves (4) if, and only if, (x^{diff}, u, y) with $x^{\text{diff}}(t_0^-) = \Pi x_0$ solves

$$\begin{aligned} \dot{x}^{\text{diff}} &= A^{\text{diff}} x^{\text{diff}} + B^{\text{diff}} u \\ \bar{y} &= Cx^{\text{diff}} + (D - CB^{\text{imp}})u \end{aligned} \tag{10}$$

which shows that the minmization of (2) subject to (4) is equivalent to the minimization of

$$\bar{J}(x^{\text{diff}}, u) = \int_{t_0}^{t_f} \bar{y}(t)^\top \bar{y}(t)\, \mathrm{d}t + \bar{x}(t_f^-)^\top P \bar{x}(t_f). \tag{11}$$

where $\bar{x} := x^{\text{diff}} - B^{\text{imp}}u$, subject to (10). This shows that the optimal control problem for non switched DAEs can be reduced to an equivalent problem for ordinary differential equations (ODEs). However, note that the latter problem is still not a standard finite horizon linear quadratic optimal control problem for ODEs as the terminal state of the input is penalized by the terminal cost and because of the subspace endpoint constraint $\bar{x}(t_f^-) = x(t_f^-) \in \mathcal{V}^{\text{end}}$.

In order to ensure that the optimal input does not contain impulses, we assume that $\bar{D} := D - CB^{\text{imp}}$ has full column rank, such that $\bar{D}^\top \bar{D}$ is positive definite. For ODE optimal control problems with a cost resulting from an output (10) this is standard. However, in the literature on optimal control on DAE it is often assumed that $D^\top D$ is positive definite, which we do not require here. Note that a sufficient condition for $\bar{D}$ to have full column rank is

$$\operatorname{rank}\,[CW \;\; D] = m$$

were W is some projector onto $\ker E$. This assumption is very similar to the assumption that the system (4) is impulse-observable.

3.1 *Regarding the terminal cost*

As the terminal cost in (11) penalizes the input, it follows that if there exists a solution that minimizes (11), the value of $u(t_f^-)$ must be well defined and satisfies given the optimal state $x^{\text{diff}}(t_f^-)$

$$\begin{aligned} &((x^{\text{diff}}(t_f^-) - B^{\text{imp}}u(t_f^-))^\top P((x^{\text{diff}}(t_f^-) - B^{\text{imp}}u(t_f^-)) \\ &\quad \leqslant ((x^{\text{diff}}(t_f^-) - B^{\text{imp}}v)^\top P((x^{\text{diff}}(t_f^-) - B^{\text{imp}}v)) \end{aligned}$$

for all v satisfying $x^{\text{diff}}(t_f^-) - B^{\text{imp}}v \in \mathcal{V}^{\text{end}}$. Using Lagrange multipliers and noting that the terminal cost is a convex function of $u(t_f^-)$ leads to the following result.

Lemma 7. Let (x^{diff}, u) be a solution satisfying $\bar{x}(t_0^-) = x_0$ and $\bar{x}(t_f^-) \in \mathcal{V}^{\text{end}}$ that minimizes (11). Then the terminal cost satisfies

$$\bar{x}(t_f^-)^\top P \bar{x}(t_f^-) = x^{\text{diff}}(t_f^-)^\top \Psi^\top P \Psi x^{\text{diff}}(t_f^-). \tag{12}$$

where $\Psi = (I - B^{\text{imp}}N)$ for some N satisfying

$$[\,I \;\; 0 \;\; N\,] \ker \begin{bmatrix} B^{\text{imp}\top} P B^{\text{imp}} & B^{\text{imp}\top} \Pi_{\mathcal{V}^\perp}^\top & -2B^{\text{imp}\top} P \Pi \\ \Pi_{\mathcal{V}^\perp} B^{\text{imp}} & 0 & -\Pi_{\mathcal{V}^\perp} \Pi \end{bmatrix} = 0.$$

where $\Pi_{\mathcal{V}^\perp}$ is an orthogonal projector onto the orthogonal complement of $\mathcal{V}^{\text{end}}$.

It follows from Lemma 7 that instead of minimizing (11) directly, we can focus on finding an input that minimizes

$$\begin{aligned} \bar{J}_\Psi(x^{\text{diff}}, u) = &\int_{t_0}^{t_f} \bar{y}(t)\bar{y}(t)\, \mathrm{d}t \\ &+ x^{\text{diff}}(t_f^-)^\top \Psi^\top P \Psi x^{\text{diff}}(t_f^-) \end{aligned} \tag{13}$$

and verify whether the optimal input satisfies (12). However, the computation of the input that minimizes (13) is rather straightforward. After denoting

$$\bar{y}(t)^\top \bar{y}(t) = \begin{bmatrix} x^{\text{diff}} \\ u \end{bmatrix}^\top \begin{bmatrix} Q & S^\top \\ S & R \end{bmatrix} \begin{bmatrix} x^{\text{diff}} \\ u \end{bmatrix}$$

we can state the following result.

Lemma 8. The cost functional $\bar{J}_\Psi(x^{\text{diff}}, u)$ satisfies

$$\begin{aligned} &\bar{J}_\Psi(x^{\text{diff}}, u) - x^{\text{diff}}(t_0^-)^\top X(t_0) x^{\text{diff}}(t_0^-) \\ &\quad = \int_{t_0}^{t_f} \left(\left\| u + R^{-1}(\bar{B}^{\text{diff}\top} X + S^\top)\bar{x}^{\text{diff}} \right\|_2^2 \right) \end{aligned}$$

where X solves

$$\begin{aligned} \dot{X} = &A^{\text{diff}\top} X + X^\top A^{\text{diff}} + Q \\ &- (S + X^\top B^{\text{diff}}) R^{-1} (B^{\text{diff}\top} X + S^\top). \end{aligned} \tag{14}$$

with terminal condition $X(t_f) = \Psi^\top P \Psi$.

Corollary 9. If an input u minimizes (13), then

$$u = -R^{-1}(B^{\text{diff}\top} X + S^\top) x^{\text{diff}} \tag{15}$$

where X is a solution to (14) with $X(t_f) = \Psi^\top P \Psi$.

The result of Corollary 9 shows that if there exists an optimal control, it needs to be of a particular form. However, a feedback of the form (15) does not necessarily controls an initial value Πx_0 to the desired subspace, *e.g.*, in the case $\mathcal{V}^{\text{end}}$ is the zero subspace, and hence an optimal control might fail to exist. In order to determine which intial values are controlled to $\mathcal{V}^{\text{end}}$ at t_f^- we define the following flow operator.

Definition 10. The backwards state transition matrix for the closed loop ODE

$$\dot{x}^{\text{diff}} = \left(A^{\text{diff}} - B^{\text{diff}} R^{-1} \left(B^{\text{diff}\top} X + S^\top\right)\right) x^{\text{diff}}$$

is given by $\Omega(t, t_f)$. Hence $x^{\text{diff}}(t) = \Omega(t, t_f) x^{\text{diff}}(t_f^-)$.

Recall that the state $x = x^{\text{diff}} - B^{\text{imp}} u$ and thus for the input (15) we have $x(t_f^-) = M x^{\text{diff}}(t_f^-)$ where

$$M := I - B^{\text{imp}} R^{-1} \left(B^{\text{diff}\top} \Psi^\top P \Psi + S^\top\right).$$

As $x^{\text{diff}}(t_f^-) = \Pi\xi$ for some $\xi \in \mathbb{R}^n$ it follows that $x(t_f^-) \in \mathcal{V}^{\text{end}}$ if and only if

$$x^{\text{diff}}(t_f^-) = \Pi\xi \in \ker \Pi_{\mathcal{V}^\perp} M$$

Next, observe that the input (15) satisfies (12) if and only if

$$\begin{aligned} x(t_f^-)^\top P x(t_f^-) &= x^{\text{diff}}(t_f^-)^\top M^\top P M \Pi x(t_f^-) \\ &= x^{\text{diff}}(t_f^-)^\top \Psi^\top P \Psi x^{\text{diff}}(t_f^-). \end{aligned}$$

This is the case if $M^\top P M x^{\text{diff}}(t_f^-) = \Psi^\top P \Psi x^{\text{diff}}(t_f^-)$. Given these observations, we can state the following result regarding the minimization of (11).

Theorem 11. There exists an impulse-free solution (x, u) satisfying $x(t_0^-) = x_0$ and $x(t_f^-) \in \mathcal{V}^{\text{end}}$ that minimizes (11) if and only if

$$x_0 \in \mathcal{V}^{\text{init}} := \Omega(t_0, t_f) \ker \begin{bmatrix} \Pi_{\mathcal{V}^\perp} M \\ M^\top P M - \Psi^\top P \Psi \end{bmatrix} \Pi.$$

3.2 Multiple switched case

Given the result of Theorem 11 we are now able to state conditions for the existence of a solution that minimizes (2) subject to (1). To do so, we define the following sequence

$$\begin{aligned} \mathcal{V}_{\mathtt{n}}^{\text{end}} &= \mathcal{V}^{\text{end}}, \\ \mathcal{V}_{i-1}^{\text{end}} &= \mathcal{V}_i^{\text{init}}, \qquad i = \mathtt{n}, \mathtt{n}-1, ..., 0 \end{aligned}$$

where $\mathcal{V}_i^{\text{init}}$ is defined according to Theorem 11 on the interval $[t_i, t_{i+1})$ w.r.t. $\mathcal{V}_i^{\text{end}}$.

Theorem 12. There exists an impulse-free solution (x, u) that minizes (2) and satisfies $x(t_0^-) = x_0$ and $x(t_f^-) \in \mathcal{V}^{\text{end}}$ if and only if $x_0 \in \mathcal{V}_0^{\text{init}}$.

4. CONCLUSION

In this abstract we considered the finite horizon optimal control problem for switched DAEs. Based on sovability of $\mathtt{n}$ nonstandard optimal control problems for ODEs solvability of the optimal control problem for switched DAEs can be concluded. In this abstract impulse-freeness of the solution (x, u) was required. A natural direction of research is to investigate the existence of optimal impulsive solutions.

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