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A time-varying approach for model reduction of singular linear switched systems in discrete time

Md. Sumon Hossain [*] **Sutrisno** [*,**] **Stephan Trenn** [*]

[*] *Bernoulli Institute for Mathematics, Computer Science, and Artificial Intelligence, University of Groningen, The Netherlands.*
[**] *Dept. of Mathematics, Diponegoro University, Indonesia.*

Abstract: We propose a model reduction approach for singular linear switched systems in discrete time with a fixed mode sequence based on a balanced truncation reduction method for linear time-varying discrete-time systems. The key idea is to use the one-step map to find an equivalent time-varying system with an identical input-output behavior, and then adapt available balance truncation methods for (discrete) time-varying systems. The proposed method is illustrated with a low-dimensional academic example.

Keywords: singular linear switched systems, time-varying systems, reachability and observability Gramians and balanced truncation.

1. INTRODUCTION

In this paper we consider singular linear switched systems (SLSSs) in discrete time of the form

$$S_\sigma : \begin{cases} E_{\sigma(k)}x(k+1) = A_{\sigma(k)}x(k) + B_{\sigma(k)}u(k), \\ \qquad y(k) = C_{\sigma(k)}x(k),\ k \in \mathbb{N}, \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state at time $k \in \mathbb{N}$ and $\sigma : \mathbb{N} \to Q = \{0, 1, 2, \cdots, \mathtt{m}\}$, $\mathtt{m} \in \mathbb{N}$, is the switching signal with the switching times $0 < s_1 < s_2 < \cdots < s_\mathtt{m}$ in the bounded interval $[k_0, k_f) := \{k_0, k_0 + 1, \ldots, k_f - 1\}$ of interest. The system matrices are $E_i, A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $C_i \in \mathbb{R}^{p\times n}$, where $i \in Q$. The matrices E_i are in general singular, which is related to the presence of (mode-dependent) algebraic constraints. We assume that the i-th mode is active in the interval $[s_i, s_{i+1})$, for $i = 0, 1, \cdots, \mathtt{m}$ (where $s_0 := 0$) and define the duration of the i-th mode as $\tau_i = s_{i+1} - s_i$. Since we will be interested in the input-output behavior of S_σ we assume in the following that $x(0) = 0$.

Control problems governed by SLSSs arise in a variety of practical applications including circuit simulation, computational electromagnetics, fluid dynamics, mechanical and chemical engineering; see Luenberger (1977); Xia et al. (2008). In some cases, these systems lead to analyzing large-scale and complex dynamical systems. Although, the computational speed and performance of the modern computers are increasing; simulation, optimization or real time controller design for such large-scale systems are still difficult due to extra memory requirements and additional computational complexity. Model order reduction (MOR) is a useful tool for dealing with such complexity, wherein one seeks a simpler model that can then be used as an efficient surrogate model to the original model. There are already some existing results on MOR for switched ODEs, see e.g. Schulze and Unger (2018); Gosea et al. (2020) for continuous time case, and Baştuğ et al. (2016, 2014); Shaker and Wisniewski (2012); Birouche et al. (2012) for discrete time case. However, in contrast to the existing literature, we view here the SLSS (1) as a *time-varying* linear systems, in particular, the reduction in general depends on the specifically given switching signal and results in a time-varying reduced model.

The remaining paper is structured as follows. We discuss the problem formulation and some preliminaries for singular system in Section 2. Section 3 provides the computation procedure of time-varying balanced realization in discrete time. In Section 4, we present time-varying balanced truncation method for SLSS. Finally, some numerical results are presented in Section 5.

2. PRELIMINARIES AND PROBLEM STATEMENT

In this section, it is shown that the solutions of a SLSS can equivalently be expressed in terms of a time-varying system. For the existence and uniqueness of solutions of SLSSs the following assumption is needed.

Assumption 1. The SLSS (1) is *jointly index-1*, i.e.

$$\mathcal{S}_i \oplus \ker E_j = \mathbb{R}^n, \quad \forall i, j \in Q,$$

where $\mathcal{S}_i = A_i^{-1}(\operatorname{im} E_i)$. ◇

Under the jointly index-1 assumption, the solution of SLSS (1) with $x(0) = 0$ exists. This solution is unique and satisfies the following lemma.

Lemma 1. (Cf. Anh et al. (2019)) Assume the SLSS (1) is jointly index-1. For a given switching signal σ, there exist corresponding matrices $\widetilde{A}_k$, $\widetilde{B}_k$ and $\widetilde{F}_k$, such that all solutions of (1) with $x(0) = 0$ satisfy

$$x(k+1) = \widetilde{A}_k x(k) + \widetilde{B}_k u(k) + \widetilde{F}_k u(k+1), \quad k \in \mathbb{N}. \tag{2}$$

⋆ Email Addresses: s.hossain@rug.nl; s.sutrisno[@rug.nl, @live.undip.ac.id]; s.trenn@rug.nl;

Proof. Let $\sigma(-1) := \sigma(0)$ and, for $k \in \mathbb{N}$,

$$\widetilde{A}_k := V_{\sigma(k)} \begin{bmatrix} \bar{A}^1_{\sigma(k),\sigma(k-1)} & 0 \\ -\bar{A}^2_{\sigma(k+1),\sigma(k)}\bar{A}^1_{\sigma(k),\sigma(k-1)} & 0 \end{bmatrix} V^{-1}_{\sigma(k-1)}, \quad (3a)$$

$$\widetilde{B}_k := V_{\sigma(k)} \begin{bmatrix} \bar{B}^1_{\sigma(k),\sigma(k-1)} \\ -\bar{A}^2_{\sigma(k+1),\sigma(k)}\bar{B}^1_{\sigma(k),\sigma(k-1)} \end{bmatrix}, \quad (3b)$$

$$\widetilde{F}_k := V_{\sigma(k)} \begin{bmatrix} 0 \\ -\bar{B}^2_{\sigma(k+1),\sigma(k)} \end{bmatrix}, \quad (3c)$$

where

$$\begin{bmatrix} \bar{A}^1_{i,j} & 0 \\ -\bar{A}^2_{i,j} & I_{n_2} \end{bmatrix} = V_i^{-1} G^{-1}_{i,j} A_i V_j, \quad \begin{bmatrix} \bar{B}^1_{i,j} \\ \bar{B}^2_{i,j} \end{bmatrix} = V_i^{-1} G^{-1}_{i,j} B_i,$$

$$G_{i,j} = E_i + A_i Q_{i,j}, \quad Q_{i,j} = V_j \begin{bmatrix} 0 & 0 \\ 0 & I_{n_2} \end{bmatrix} V_i^{-1},$$

$V_i = [g_i^1, ..., g_i^{n_1}, h_i^{n_1+1}, ..., h_i^n]$, $g_i^1, ..., g_i^{n_1}$ are the bases of $\mathcal{S}_i$, and $h_i^{n_1+1}, ..., h_i^n$ are the bases of $\ker E_i$. The remaining proof is similar to the proof of (Anh et al., 2019, Thm. 5.1) and therefore omitted. ■

Remark 2. The one-step map from $x(k)$ to $x(k+1)$ depends on the modes at time $k-1$, k and $k+1$. This concludes that the allowed space of consistent initial values also depends on the choice of $\sigma(-1)$, here we assume that $\sigma(-1) = \sigma(0)$. As pointed out in (Anh et al., 2019, Rem. 5.2), the effect of a different choice $\sigma(-1)$ is not yet fully understood, and is still under investigation; nevertheless, since we restrict our attention to the initial condition $x(0) = 0$, this is of no further concern to us here.

Motivated by Lemma 1, we consider the following time-varying surrogate system for (1) with given switching signal σ:

$$\widetilde{S}_\sigma : \begin{cases} x(k+1) = \widetilde{A}_k x(k) + \begin{bmatrix} \widetilde{B}_k & \widetilde{F}_k \end{bmatrix} \widetilde{u}(k), \\ y(k) = C_k x(k), \ k \in \mathbb{N}, \end{cases} \quad (4)$$

where $x(0) = 0$, $\widetilde{u}(k) = \begin{bmatrix} u(k) \\ u(k+1) \end{bmatrix}$, $C_k := C_{\sigma(k)}$ and $\widetilde{A}_k$, $\widetilde{B}_k$, $\widetilde{F}_k$ are given by (3). Writing $\tilde{u} = \begin{bmatrix} I \\ \mathcal{T}_1 \end{bmatrix} u$, where $\mathcal{T}_1\{u\}(k) := u(k+1)$ denotes the time-shift operator, by Lemma 1, (1) and (4) have the same input-output behaviour.

Note that the solution of jointly index-1 SLSS (1) does not exist for any initial value $x(0) \in \mathbb{R}^n$. In fact, the consistency space of jointly index-1 (1), under the assumption $\sigma(-1) = \sigma(0)$, is $\operatorname{im} V_{\sigma(0)} \begin{bmatrix} I & 0 \\ -\widehat{A}^2_{\sigma(0),\sigma(0)} & \widehat{B}^2_{\sigma(0),\sigma(0)} \end{bmatrix}$. This has some implications on the relationship between system S_σ and $\widetilde{S}_\sigma$ in terms of observability and reachability. Here, observability means that if the input and output are identically zero on $[k_0, k_f)$ also the state has to be zero; reachability means, that for each $x_f \in \mathbb{R}^n$, there exists an input such that the corresponding solution satisfies $x(k_f - 1) = x_f$. Clearly, a reachable SLSS S_σ implies a reachable time varying surrogate system $\widetilde{S}_\sigma$ whereas an observable SLSS S_σ does not imply that its surrogate system $\widetilde{S}_\sigma$ is observable. However, an observable $\widetilde{S}_\sigma$ implies that S_σ is also observable.

Our goal is to find for the time-varying system (4) a reduced size time-varying system

$$\widehat{S}_\sigma : \begin{cases} \widehat{x}(k+1) = \widehat{A}_k \widehat{x}(k) + \begin{bmatrix} \widehat{B}_k & \widehat{F}_k \end{bmatrix} \begin{bmatrix} u(k) \\ u(k+1) \end{bmatrix}, \\ \widehat{y}(k) = \widehat{C}_k \widehat{x}(k), \ k \in \mathbb{N}. \end{cases} \quad (5)$$

with reduced system matrices $\widehat{A}_i \in \mathbb{R}^{r \times r}$, $\widehat{B}_i, \widehat{F}_i \in \mathbb{R}^{r \times m}$, $\widehat{C}_i \in \mathbb{R}^{p \times r}$ and $r \ll n$, such that $\widehat{y} \approx y$ for a large class of inputs u. Due to the input-output equivalence between (1) and (4), the reduced system (5) will then also be good surrogate model for the original SLSS.

3. TIME-VARYING BALANCED REALIZATIONS

3.1 Time-varying Gramians

Consider a time-varying discrete time system of the form

$$\begin{aligned} x(k+1) &= A_k x(k) + B_k u(k), \quad k \in [k_0, k_f) \\ y(k) &= C_k x(k). \end{aligned} \quad (6)$$

Definition 3. The time-varying reachability and observability Gramians of (6) are defined recursively as

$$P(k) = A_{k-1} P(k-1) A^\top_{k-1} + B_{k-1} B^\top_{k-1}, \quad (7)$$

$$Q(k) = A^\top_k Q(k+1) A_k + C^\top_k C_k, \quad (8)$$

with some positive semi-definite initial/final values $P(k_0) = P_0$ and $Q(k_f) = Q_f$

Note that the reachability Gramian is constructed *forward* in time, while the observability Gramians evolves *backward* in time.

Remark 4. The choice of the initial/final Gramians is crucial in the sense that they play an important role for the magnitude of all other subsequent Gramians. At this moment the best choice of the initial/final Gramians is not clear. In the context of time-varying case, two versions can be proposed for the initial/final Gramians. One choice could be to assume that the first mode is active in the past, i.e. $(-\infty, k_0]$, and the Gramians of the first mode is considered as the initial reachability Gramian. Similarly, by assuming that the last mode will be active in the future, i.e. $[k_f, \infty)$, and the Gramian of the last is considered as the final value for observability Gramian. However, in this choice, the computation of infinite Gramians is only possible for stable modes; here, we do not assume stability of each mode. On the other hand, a second choice could be the identity matrix which would not affect the direction of the states which are difficult to control and difficult to observe. By scaling the identity matrix with a smaller magnitude, one can restrict the influence of these artificial initial/final Gramians relatively to the time-varying Gramians and also for the bounded time-varying coordinate transformation matrices.

Note that, $P(k)$ and $Q(k)$ are both symmetric and positive semidefinite for all $k \in [k_0, k_f]$ if P_0 and Q_f are positive definite. It is assumed that the input-output balancing with respect to the reachability and observability Gramians is defined over specific time intervals. Hence, no assumption is needed with regard to the stability of the system.

Applying any time-varying coordinate transformation

$$x(k) = T(k)\overline{x}(k)$$

to (6) results in an equivalent system

$$\overline{x}(k+1) = \overline{A}_k \overline{x}(k) + \overline{B}_k u(k)$$
$$y(k) = \overline{C}_k x(k),$$

with $\overline{A}_k := T(k+1)^{-1} A_k T(k)$, $\overline{B}_k := T(k+1)^{-1} B_k$, $\overline{C}_k := C_k T(k)$. It is easily seen, the corresponding Gramians satisfy

$$\overline{P}(k) = T(k)^{-1} P(k) T(k)^{-\top},$$
$$\overline{Q}(k) = T(k)^{\top} Q(k) T(k),$$

if the initial/final values satisfy $\overline{P}_0 = T(0)^{-1} P_0 T(0)^{-\top}$ and $\overline{Q}_f = T(k_f)^{\top} Q_f T(k_f)$. In particular,

$$\overline{P}(k)\overline{Q}(k) = T(k)^{-1} P(k) Q(k) T(k).$$

This shows that, under such transformation, the eigenvalues of the product of Gramians are invariant.

The key idea of balanced truncation is to find a coordinate transformation such that the corresponding Gramians become equal and diagonal. How to achieve such a balancing transformation is given in the following lemma.

Lemma 5. Assume that Gramiens $P(k)$ and $Q(k)$ of the time-varying system (6) are nonsingular on $[k_0, k_f)$. Then, there exists a transformation $T : [k_0, k_f] \to \mathbb{R}^{n \times n}$ such that

$$T(k)^{-1} P(k) T(k)^{-\top} = T(k)^{\top} Q(k) T(k) = \Xi(k), \qquad (9)$$

for all $k \in [k_0, k_f]$ and $\Xi(k) = \{\xi_1(k), \dots, \xi_n(k)\}$ is a diagonal matrix. In fact, the transformation matrices are given by

$$T(k) = R(k) U(k) \Xi(k)^{-1/2},$$
$$T(k)^{-1} = \Xi(k)^{-1/2} V(k)^{\top} L(k)^{\top},$$

where $U(k)\Xi(k)V(k)^{\top}$ is the singular value decomposition of $R(k)^{\top} L(k)$, and where $R(k)R(k)^{\top} = P(k)$ and $L(k)L(k)^{\top} = Q(k)$ are the Cholesky decompositions of P and Q, respectively.

Proof. The proof is similar to the proof of (Hossain and Trenn, 2020, Lemma 11) and therefore omitted. ■

4. MODEL REDUCTION

We now combine the above results to propose a model reduction method for SLSS (1) based on balanced truncation. By Assumption 1, we can instead consider system (4) and we can construct the corresponding time-varying reachability/observability Gramians $\widetilde{P}(k)$ and $\widetilde{Q}(k)$ for $(\widetilde{A}_k, [\widetilde{B}_k, \widetilde{F}_k], \widetilde{C}_k)$ for some initial/final Gramians $\widetilde{P}_0, \widetilde{Q}_f$. Now an assumption is needed for model reduction methods.

Assumption 2. Assume a transformation $\widetilde{T}$ such that the balanced Gramians are obtained by

$$\widetilde{T}(k)^{-1} \widetilde{P}(k) \widetilde{T}(k)^{-\top} = \widetilde{T}(k)^{\top} \widetilde{Q}(k) \widetilde{T}(k) = \widetilde{\Xi}(k)$$

and let, the (uniformly) partitioned form $\widetilde{\Xi}(k) = \begin{bmatrix} \widehat{\Xi}(k) & 0 \\ 0 & \overline{\Xi}(k) \end{bmatrix}$, where all diagonal entries in $\overline{\Xi}(k)$ are significantly smaller than those in $\widehat{\Xi}(k)$. ◇

With the Assumption 2, the singular value decomposition is then given by

$$\widetilde{R}(k)^{\top} \widetilde{L}(k) = \begin{bmatrix} \widehat{U}(k) & \overline{U}(k) \end{bmatrix} \begin{bmatrix} \widehat{\Xi}(k) & 0 \\ 0 & \overline{\Xi}(k) \end{bmatrix} \begin{bmatrix} \widehat{V}(k) & \overline{V}(k) \end{bmatrix}^{\top}$$

where $\widetilde{R}(k)\widetilde{R}(k)^{\top} = \widetilde{P}(k)$ and $\widetilde{L}(k)\widetilde{L}(k)^{\top} = \widetilde{Q}(k)$ are obtained by a Cholesky decomposition. According to this splitting, let $\widetilde{T}(k) = [\widehat{\Pi}_R(k), *]$ and $\widetilde{T}(k)^{-1} = [\widehat{\Pi}_L(k), *]^{\top}$, and define

$$\widehat{A}_k := \widehat{\Pi}_L(k+1) \widetilde{A}_k \widehat{\Pi}_R(k),$$
$$\begin{bmatrix} \widehat{B}_k & \widehat{F}_k \end{bmatrix} := \widehat{\Pi}_L(k+1) \begin{bmatrix} \widetilde{B}_k & \widetilde{F}_k \end{bmatrix},$$
$$\widehat{C}_k := \widetilde{C}_k \widehat{\Pi}_R(k),$$

which results in our proposed reduced system (5), where the left- and right-projectors are calculated as

$$\widehat{\Pi}_R(k) := \widetilde{R}(k) \widehat{U}(k) \widehat{\Xi}(k)^{-1/2} \in \mathbb{R}^{n \times r},$$
$$\widehat{\Pi}_L(k) := \widehat{\Xi}(k)^{-1/2} \widehat{V}(k)^{\top} \widetilde{L}(k)^{\top} \in \mathbb{R}^{r \times n}.$$

5. NUMERICAL RESULTS

This section illustrates the proposed method by providing an example.

Example 6. Consider a SLSS with two modes

$$(E_0, A_0, B_0, C_0) = \left(\begin{bmatrix} 1&0&0&0 \\ 1&1&0&0 \\ 0&0&1&0 \\ 0&0&0&0 \end{bmatrix}, \begin{bmatrix} 1&0&0&0 \\ 1&0&1&0 \\ 1&1&1&0 \\ 0&0&0&1 \end{bmatrix}, \begin{bmatrix} 0.02 \\ 2 \\ 1 \\ 0.2 \end{bmatrix}, \begin{bmatrix} -0.1 \\ 0.1 \\ 0.1 \\ 2 \end{bmatrix}^{\top} \right),$$

$$(E_1, A_1, B_1, C_1) = \left(\begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&0 \end{bmatrix}, \begin{bmatrix} 1&0&0&0 \\ 0&1&1&0 \\ 1&0&1&0 \\ 0&0&0&1 \end{bmatrix}, \begin{bmatrix} 0.01 \\ 2 \\ 0.5 \\ 0.1 \end{bmatrix}, C_0 \right),$$

Consider a switching signal $\sigma : [0, 9) \to \{0, 1\}$,

$$\sigma(k) = \begin{cases} 0 & : k \in [0,4) \cup [7,9), \\ 1 & : k \in [4,7). \end{cases}$$

It can easily be verified that the pairs (E_0, A_0) and (E_1, A_1) are jointly index-1. Hence, by Lemma 1, the time-varying system (4) is obtained with the following system matrices

$$(\widetilde{A}_k, \widetilde{B}_k) = \begin{cases} \left(\begin{bmatrix} 1&0&0&0 \\ 0&0&1&0 \\ 1&1&1&0 \\ 0&0&0&0 \end{bmatrix}, \begin{bmatrix} 0.02 \\ 1.98 \\ 1 \\ 0 \end{bmatrix} \right) & : k = 0,1,2,3,7,8, \\ \left(\begin{bmatrix} 1&0&0&0 \\ 0&1&1&0 \\ 1&0&1&0 \\ 0&0&0&0 \end{bmatrix}, \begin{bmatrix} 0.01 \\ 2 \\ 0.5 \\ 0 \end{bmatrix} \right) & : k = 4,5,6, \end{cases}$$

$$\widetilde{F}_k = \begin{cases} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.2 \end{bmatrix} & : k = 0,1,2,6,7,8, \\ \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.1 \end{bmatrix} & : k = 3,4,5. \end{cases}$$

The corresponding reachability and observability Gramians are calculated respectively, $\widetilde{P}(k)$ and $\widetilde{Q}(k)$ for $k \in [0, 9)$ with initial/final values $\widetilde{P}_0 = 0.002I$ and $\widetilde{Q}_f = 0.002I$. The corresponding HSVs are depicted in Figure 1 and it is apparent that the last two HSVs are significantly smaller than the first two. Hence, a two dimensional reduced system is obtained which approximates the time-varying system (4) and hence, the original SLSS.

The computed two dimensional reduced systems at each time steps are given by $(\widehat{A}_k, [\widehat{B}_k, \widehat{F}_k], \widehat{C}_k) =$

$$\left(\begin{bmatrix} 0.9206 & -0.0051 \\ -0.0107 & 0.0012 \end{bmatrix} \begin{bmatrix} -1.8615 & 0.0046 \\ -0.0535 & 0.6305 \end{bmatrix} \begin{bmatrix} -0.1410 \\ -0.6334 \end{bmatrix}^{\top} \right),$$

$$\left(\begin{bmatrix} 0.9761 & -0.0071 \\ -0.0058 & -0.0076 \end{bmatrix} \begin{bmatrix} -1.0832 & 0.0074 \\ -0.0603 & 0.6287 \end{bmatrix} \begin{bmatrix} -0.2387 \\ -0.6332 \end{bmatrix}^{\top} \right),$$

$$\left(\begin{bmatrix} 0.9887 & -0.0116 \\ -0.0027 & -0.0071 \end{bmatrix} \begin{bmatrix} -0.7265 & 0.0117 \\ -0.0445 & 0.8861 \end{bmatrix} \begin{bmatrix} -0.3859 \\ -0.4449 \end{bmatrix}^{\top} \right),$$

$$\left(\begin{bmatrix} 1.1336 & -0.0155 \\ -0.4762 & -0.0398 \end{bmatrix} \begin{bmatrix} -0.3934 & 0.0143 \\ -0.0311 & 0.4016 \end{bmatrix} \begin{bmatrix} -0.7193 \\ -0.4515 \end{bmatrix}^\top\right),$$
$$\left(\begin{bmatrix} 0.9856 & -0.0335 \\ -0.0019 & 0.1912 \end{bmatrix} \begin{bmatrix} -0.2249 & 0.0206 \\ -0.0536 & 0.4158 \end{bmatrix} \begin{bmatrix} -1.0114 \\ -0.4262 \end{bmatrix}^\top\right),$$
$$\left(\begin{bmatrix} 0.9719 & -0.0475 \\ -0.0036 & 0.0701 \end{bmatrix} \begin{bmatrix} -0.1850 & 0.0286 \\ -0.0047 & 0.2986 \end{bmatrix} \begin{bmatrix} -1.3323 \\ -0.5423 \end{bmatrix}^\top\right),$$
$$\left(\begin{bmatrix} 0.8014 & -0.0770 \\ -0.8119 & 0.0689 \end{bmatrix} \begin{bmatrix} -0.1249 & 0.0523 \\ 0.1196 & -0.5938 \end{bmatrix} \begin{bmatrix} -1.4777 \\ 0.5432 \end{bmatrix}^\top\right),$$
$$\left(\begin{bmatrix} 0.9471 & 0.0834 \\ -0.0047 & 0.0125 \end{bmatrix} \begin{bmatrix} -0.1026 & 0.1118 \\ -0.0261 & 0.4245 \end{bmatrix} \begin{bmatrix} -2.8391 \\ -0.1943 \end{bmatrix}^\top\right).$$

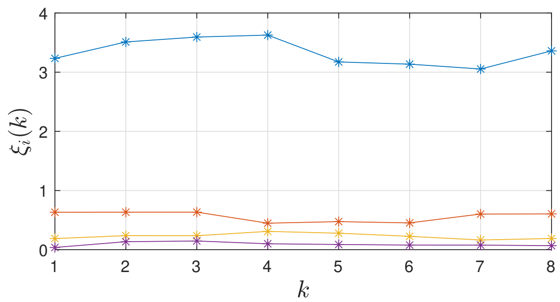

Fig. 1. Hankel singular values of balanced Gramians at each time instance.

Consider randomly generated input $u(\cdot)$ with $u(0) = 0$, and the input-output behavior is calculated for the system (4) and its reduced system with relative errors. Figure 2 displays the output, the input signal, and the relative error for the original system and the proposed two dimensional reduced system. Clearly, both outputs match nicely and the relative error is less then 6%.

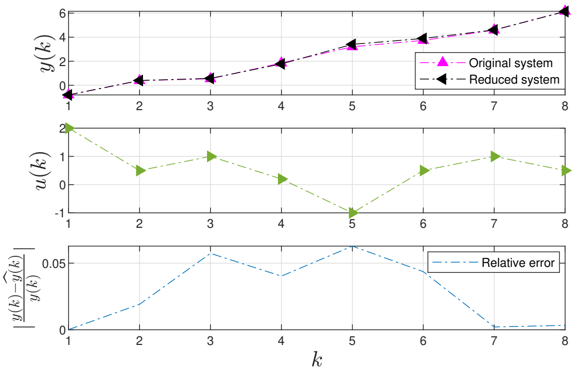

Fig. 2. Outputs and the relative error of the original system (4) and the proposed 2^{nd} order approximation.

Next, another initial /final value of the Gramians is considered by increasing the magnitudes as $\widetilde{P}_0 = 0.5I$ and $\widetilde{Q}_f = 0.5I$. With the same input sequence as in Figure 2, the input-output behavior with the relative error is depicted in Figure 3, which shows that the choice of the initial /final values of Gramians plays an important role in the error analysis. Therefore, it is concluded that taking small magnitude with identity matrix could be the best choice for the initial /final values of the Gramians.

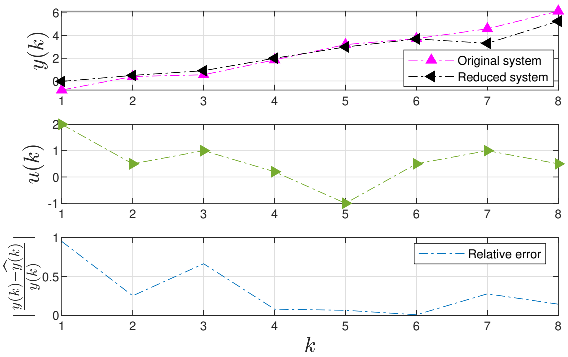

Fig. 3. Outputs and relative errors of the original system (4) and the proposed 2nd order approximation with initial/final values $\widetilde{P}_0 = 0.5I$, $\widetilde{Q}_f = 0.5I$.

6. CONCLUSION

In this paper, we have presented a time-varying approach for proposing a reduced system for singular linear switched systems. The key novelty is the viewpoint of the SLSS as a piecewise-constant time-varying system. At first, we have focused on input-extended time-varying ODEs, which gives identical input-output behavior as the original index-1 SLSSs. Then, by applying the well known time-varying balanced truncation method for the discrete time case, we find a good approximation of the time-varying system.

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