Impulse-controllability of system classes of switched DAEs

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Introduction

In this note we study system classes containing *switched differential algebraic equations* (switched DAEs) of the following form:

$$E_\sigma \dot{x} = A_\sigma x + B_\sigma u, \tag{1}$$

where $\sigma : \mathbb{R} \to \mathbb{N}$ is the switching signal and $E_p, A_p \in \mathbb{R}^{n\times n}$, $B_p \in \mathbb{R}^{n\times m}$, for $p,n,m \in \mathbb{N}$. In general, trajectories of switched DAEs exhibit jumps (or even impulses), which may exclude classical solutions from existence. Therefore, we adopt the *piecewise-smooth distributional solution framework* introduced in [1]. We study impulse controllability of system classes where a class is said to be strongly impulse controllable if every system contained in it is impulse controllable.

System Classes

In particular, we study classes of switched DAEs with a switching signal in the class $\mathcal{S}_{\mathtt{n}}$ defined as follows.

Definition 1 *The class of (arbitrary) switching signals $\mathcal{S}_{\mathtt{n}}$ is defined as the set of all $\sigma : \mathbb{R} \to \{0,1,...,\mathtt{n}\}$ of the form*

$$\sigma(t) = q_p \quad t \in [t_p, t_{p+1}) \tag{2}$$

where $\mathbf{q} := (q_0, q_1, \dots, q_{\mathtt{n}}) \in \{0,1,\dots,\mathtt{n}\}^{\mathtt{n}+1}$ is the mode sequence *of σ and $t_1 < t_2 < \dots < t_{\mathtt{n}}$ are the $\mathtt{n} \in \mathbb{N}$ switching times in $(0,\infty)$ with $t_0 := 0$ and $t_{\mathtt{n}+1} := \infty$ for notational convenience. Furthermore, for a given sequence of switching times, let $\tau_i := t_{i+1} - t_i$, $i = 0,1,\dots,\mathtt{n}-1$ such that $\tau := (\tau_0, \tau_1, \dots, \tau_{\mathtt{n}-1}) \in \mathbb{R}^{\mathtt{n}}_{>0}$, defines the sequence of (finite) mode-durations.*

Note that in the above definition, we do not exclude the situation that $q_p = q_{p+1}$ for some p, effectively leading to a switching signal with less then $\mathtt{n}$ switches. Consequently, for such a switching signal the mode duration τ is not uniquely defined, as the switching time t_{p+1} can be altered without changing the actual switching signal. Nevertheless, this does not lead to any technical problems.

Definition 2 (System classes) *For a family of matrix triplets $\{(E_p, A_p, B_p)\}_{p=0}^{\mathtt{n}}$ with regular pairs (E_p, A_p), the* system class $\Sigma_{\mathtt{n}}$ *of associated switched (regular) DAEs* (1) *under arbitrary switching is given by*

$$\Sigma_{\mathtt{n}} := \{(E_\sigma, A_\sigma, B_\sigma) \mid \sigma \in \mathcal{S}_{\mathtt{n}}\},$$

where $(E_\sigma, A_\sigma, B_\sigma)$ is understood as a triple of (piecewise-constant) time-varying matrices for each specific switching signal $\sigma : (t_0, \infty) \to \{0,1,\dots,\mathtt{n}\}$.

Equipped with the notion of a system class, we state the definition of impulse controllability of (1) based on [2] and define strong impulse controllability of a system class. Before doing so, recall that for mode i the augmented consistency space is defined as

$$\mathcal{V}_{(E_i,A_i,B_i)} := \left\{ x_0 \in \mathbb{R}^n \;\middle|\; \begin{array}{l} \exists \text{ smooth solutions } (x,u) \text{ of} \\ E_i\dot{x} = A_i x + B_i u \text{ and } x(0) = x_0 \end{array} \right\}.$$

Definition 3 (Impulse controllability) *The (individual) switched DAE $(E_\sigma, A_\sigma, B_\sigma)$ is called* impulse controllable *iff for all $x_0 \in \mathcal{V}_{[E_0,A_0,B_0]}$, there exists a solution $(x,u) \in \mathbb{D}^{n+m}_{pw\mathcal{C}^\infty}$ with $x(t_0^+) = x_0$ which is impulse free.*

The whole system class $\Sigma_{\mathtt{n}}$ associated to the family $\{(E_p, A_p, B_p)\}_{p=0}^{\mathtt{n}}$ is called strongly impulse controllable, if $(E_\sigma, A_\sigma, B_\sigma)$ is impulse controllable for all $\sigma \in \mathcal{S}_{\mathtt{n}}$.

Main result

If all systems in the system class are impulse controllable, then all the systems that are effectively single switched system. This observation leads to the following result.

Theorem 1 *Consider the system class $\Sigma_{\mathtt{n}}$ associated to $\{E_p, A_p, B_p\}_{p=0}^{\mathtt{n}}$ with corresponding (individual) consistency projectors Π_p, impulse controllable spaces $\mathcal{C}_p^{imp}$ and reachability spaces $\mathcal{R}_p$. Then $\Sigma_{\mathtt{n}}$ is strongly impulse controllable if, and only if,*

$$\mathcal{V}_{[E_i,A_i,B_i]} \subseteq \mathcal{C}_j^{imp} + \mathcal{R}_i \tag{3}$$

for all $i,j \in \{0,1,...,\mathtt{n}\}$.

The result of Theorem 1 shows that impulse controllability of a system class can be determined in finitely many steps. A future direction of reasearch is the study of system classes containing switched DAEs with a fixed mode sequence.

References

[1] Stephan Trenn. *Distributional differential algebraic equations*. PhD thesis, Institut für Mathematik, Technische Universität Ilmenau, Universitätsverlag Ilmenau, Germany, 2009.

[2] Paul Wijnbergen and Stephan Trenn. Impulse controllability of switched differential-algebraic equations. In *2020 European Control Conference (ECC)*, pages 1561–1566. IEEE, 2020.