The one-step function for discrete-time nonlinear switched singular systems

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1 Introduction

Consider the class of switched systems where each mode is a discrete-time nonlinear singular system without input of the form

$$E_{\sigma(k)}x(k+1) = F_{\sigma(k)}(x(k)), \quad (1)$$

where $k \in \mathbb{N}$ is the time instant, $x(k) \in \mathbb{R}^n$ is the state, $\sigma : \mathbb{N} \to \{0,1,2,...,\mathtt{p}\}$ is the switching signal determining which mode $\sigma(k)$ is active at time instant k, $E_i \in \mathbb{R}^{n\times n}$ are singular with a constant rank i.e. $\operatorname{rank} E_i = r < n$, and $F_i(x) = (f_{1,i}(x), f_{2,i}(x), ..., f_{n,i}(x))^\top$ are vector valued functions of nonlinear functions with $f_{j,i} : \mathbb{R}^n \longrightarrow \mathbb{R}$. Define $\mathscr{S}_i := \{x \in \mathbb{R}^n : F_i(x) \in \operatorname{im} E\}$. From basic algebra, there exist invertible matrices $S_i, T_i \in \mathbb{R}^{n\times n}$ such that $S_iE_iT_i = \left[\begin{smallmatrix} I_r & 0 \\ 0 & 0\end{smallmatrix}\right]$. By using the state transformation $T^{-1}_{\sigma(k)}x(k) = \left(\begin{smallmatrix} v(k) \\ w(k)\end{smallmatrix}\right)$, $v \in \mathbb{R}^r$, $w \in \mathbb{R}^{n-r}$, system (1) can be rewritten as

$$\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}\begin{pmatrix} v(k+1) \\ w(k+1)\end{pmatrix} = S_{\sigma(k)}F_{\sigma(k)}\left(T_{\sigma(k)}\begin{pmatrix} v(k) \\ w(k)\end{pmatrix}\right) =: \begin{pmatrix} G_{\sigma(k)}(v(k),w(k)) \\ H_{\sigma(k)}(v(k),w(k))\end{pmatrix} \quad (2)$$

Inspired by the one-step map for linear switched singular systems in [1], in this study, we formulate the one-step function for nonlinear switched singular systems under the following assumptions:

Assumption 1.1. *For each* $i \in \{0,1,...,\mathtt{p}\}$*,* $\mathscr{S}_i$ *is a subspace.*

Assumption 1.2. $\mathscr{S}_i \cap \ker E_j = \{0\}\ \forall i, j \in \{0,1,...,\mathtt{p}\}$.

Remark 1.3. Since $\mathscr{S}_i$ is a subspace, the nonlinear algebraic constraint $H_i(v,w) = 0$ is equivalent to a linear algebraic constraint. Hence, the nonlinearity appears now only on $G_i(v,w)$. However, we believe that the one-step function proposed in this study could be generalized for cases with $\mathscr{S}_i$ is not necessarily a subspace; this is our ongoing work.

2 Nonswitched Systems

We discuss in this section the solution for nonswitched cases of (1) of the form

$$Ex(k+1) = F(x(k)),\ k = 0,1,... \quad (3)$$

where $E \in \mathbb{R}^{n\times n}$ is singular. Recall $\mathscr{S} = \{x \in \mathbb{R}^n : F(x) \in \operatorname{im} E\}$, and suppose that Assumptions (1.1)-(1.2) hold.

Lemma 2.1. *System* (3) *has a solution with initial condition* $x(0) = x_0 \in \mathbb{R}^n$ *if, and only if,* $x_0 \in \mathscr{S}$*. Its solution is unique and satisfies*

$$x(k+1) = \Phi_(x(k)) = \Pi^{\ker E}_{\mathscr{S}}E^+F(x(k))\ \forall k \in \mathbb{N}. \quad (4)$$

where E^+ *is a generalized inverse of* E *and* $\Pi^{\ker E}_{\mathscr{S}}$ *is the (unique) projector onto* $\mathscr{S}$ *along* $\ker E$*.*

Proof sketch: By a state transformation as in (2), system (3) can be rewritten as

$$\begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} v(k+1) \\ w(k+1)\end{pmatrix}(k+1) = \begin{pmatrix} G(v(k),w(k)) \\ H(v(k),w(k))\end{pmatrix}$$

and by Assumption 1.2, $0 = H(v(k),w(k))$ has a solution, and thus (3) has a solution. From (3),

$$x(k+1) \in E^{-1}(F(x(k))) = \{E^+F(x(k))\} + \ker E \quad (5)$$

and $x(k+1)$ must also satisfy

$$x(k+1) \in \{x \in \mathbb{R}^n : F(x) \in \operatorname{im} E\} = \mathscr{S}. \quad (6)$$

By Assumption 1.2 and the projector lemma in [1], $x(k+1)$ satisfies (4) uniquely. □

3 Switched Systems

Based on the one-step function for nonswitched systems, we have the following theorem about the the one-step function for switched systems of the form (1).

Theorem 3.1. *System* (1) *under Assumptions 1.1-1.2 has a solution with initial condition* $x(0) = x_0 \in \mathbb{R}^n$ *if, and only if,* $x_0 \in \mathscr{S}_{\sigma(0)}$*. Its solution is unique and satisfies*

$$x(k+1) = \Phi_{\sigma(k+1),\sigma(k)}(x(k)),\ \forall k \in \mathbb{N} \quad (7)$$

where $\Phi_{i,j}$ *is the one-step function from mode-*j *to mode-*i *given by*

$$\Phi_{i,j}(x(k)) := \Pi^{\ker E_j}_{\mathscr{S}_i}E^+_jF_j(x(k)) \quad (8)$$

where E^+_j *is a generalized inverse of* E_j *and* $\Pi^{\ker E_j}_{\mathscr{S}_i}$ *is the (unique) projector onto* $\mathscr{S}_i$ *along* $\ker E_j$*.*

Proof sketch: The proof is a straightforward generalization from the proof for nonswitched systems by replacing (5)-(6) with

$$x(k+1) \in E^{-1}_j(F_j(x(k))) = \{E^+_jF_j(x(k))\} + \ker E_j,$$

$$x(k+1) \in \{x \in \mathbb{R}^n : F_i(x) \in \operatorname{im} E_i\} = \mathscr{S}_i$$

respectively. □

References

[1] Pham Ky Anh, et al. "The one-step-map for switched singular systems in discrete-time." *Proc. 58th IEEE Conf. Decision Control (CDC) 2019.*