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# Funnel control

Origin and recent advances

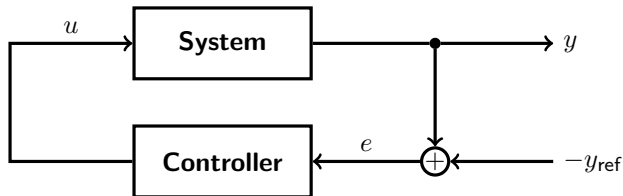
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University of Groningen, Netherlands

GRACE seminar, University of Sannio, Benevento, 31 May 2022

# Control Task

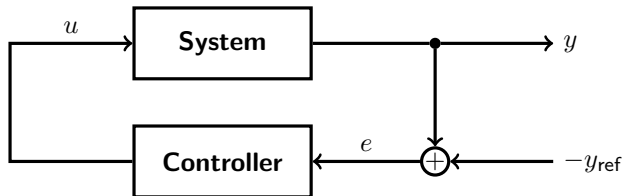


Goal: **Output tracking**  $y(t) \approx y_{\text{ref}}(t)$

## Applications

- › Flying to the moon
- › Robotics
- › (Adaptive) cruise control in cars
- › Chemical processes

# Control Task

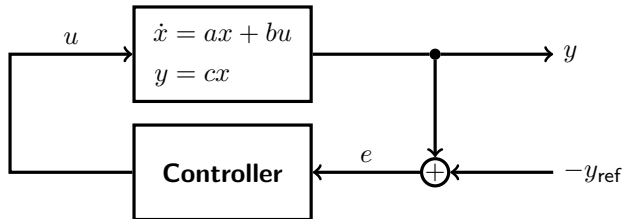


Goal: **Output tracking**  $y(t) \approx y_{\text{ref}}(t)$

## Challenge

- › no exact knowledge of system model
- › no future knowledge or model for reference signal

# The scalar linear case: Stabilization

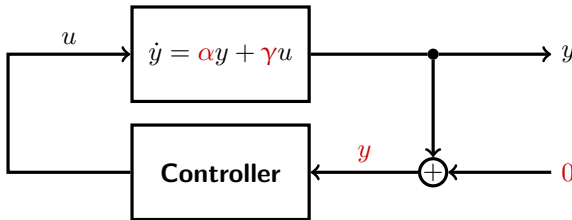


## Assumptions

- › Known model structure, in particular,  $a, b, c \in \mathbb{R}$
- › Known sign of *high frequency gain*  $\gamma := cb$ , assume  $\gamma > 0$
- ›  $y_{\text{ref}} = 0$

Unknown system parameters  $\alpha$  and  $\gamma$

# The scalar linear case: Stabilization



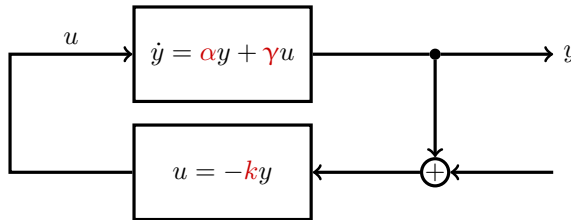
## Goal

Design feedback  $u$  (depending on  $y$ ) such that  $y(t) \rightarrow 0$  as  $t \rightarrow \infty$

If we would know  $\alpha, \gamma$ , how would we choose  $u$ ?  $\leadsto \dot{y} \stackrel{!}{=} -\lambda y$  with  $k := \frac{\alpha + \lambda}{\gamma}$

In general, with  $u = -ky$  we have  $\dot{y} = (\alpha - \gamma k)y$

# The scalar linear case: Stabilization



Hence we have arrived at our first **high gain control** result:

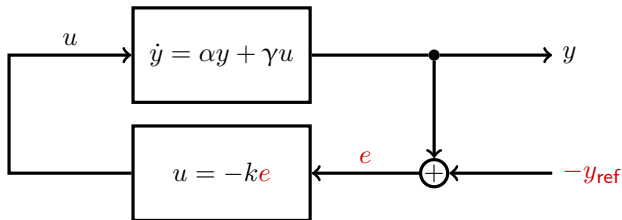
## Theorem

*The proportional negative feedback*

$$u = -ky$$

*achieves convergence for all  $k > \frac{\alpha}{\gamma}$ .*

## What happens for $y_{\text{ref}} \neq 0$ ?



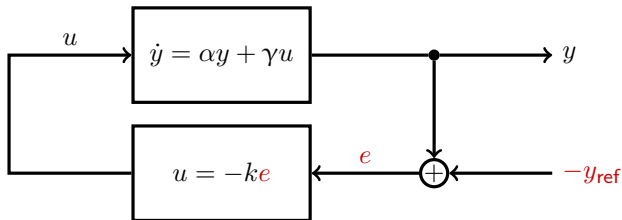
Error dynamics:  $\dot{e} = \dots = (\alpha - \gamma k)e + \alpha y_{\text{ref}} - \dot{y}_{\text{ref}}$

Equilibrium for **constant**  $y_{\text{ref}}$ :

$$0 = (\alpha - \gamma k)e + \alpha y_{\text{ref}} \quad \Longleftrightarrow \quad e = \frac{\alpha}{\gamma k - \alpha} y_{\text{ref}}$$

→ no convergence to zero anymore

## What happens for $y_{\text{ref}} \neq 0$ ?



In general: **Practical tracking** with high gain control:

## Theorem

If  $y_{\text{ref}}$  and  $\dot{y}_{\text{ref}}$  are bounded, then

$$\forall \varepsilon > 0 \exists K > 0 \forall k > K : \limsup_{t \rightarrow \infty} |e(t)| < \varepsilon$$



## Introduction

### High gain for relative degree one systems

- Linear systems

- Relative degree and zero dynamics

- High gain stabilization

- Nonlinear systems

### Adaptive choice of gain

- Adaptive stabilization

- $\lambda$ -tracking

### The funnel controller

- The original funnel controller with proof sketch

- Relative degree two funnel controller

- Bang-bang funnel control

- Funnel synchronization

## Summary



# What is the meaning of the relative degree?

## Frequency domain interpretation

Transfer function  $c(sI - A)^{-1}b =: \frac{p(s)}{q(s)}$ , then  $r = \deg(q(s)) - \deg(p(s))$

Interpretation in time-domain:

## Theorem (Byrnes-Isidori form)

$(A, b, c)$  has relative degree  $r \in \{1, \dots, n\}$  if and only if there exists a coordinate transformation  $T$  such that  $\begin{pmatrix} \eta \\ z \end{pmatrix} = Tx$  such that  $y = \eta_1, \dot{y} = \eta_2, \dots, y^{(r-1)} = \eta_r$

$$TAT^{-1} = \begin{bmatrix} 0 & 1 & & & 0 \\ & \ddots & \ddots & & \\ & & 0 & 1 & \\ a_{11} & & & & a_{12} \\ A_{21} & & & & A_{22} \end{bmatrix}, Tb = \begin{bmatrix} 0 \\ \gamma \\ 0 \end{bmatrix}, CT^{-1} = [1, 0, \dots, 0],$$

with  $a_{11} \in \mathbb{R}^{1 \times r}$ ,  $a_{12} \in \mathbb{R}^{1 \times (n-r)}$ ,  $A_{21} \in \mathbb{R}^{(n-r) \times r}$ ,  $A_{22} \in \mathbb{R}^{(n-r) \times (n-r)}$

# What is the meaning of the relative degree?

## Frequency domain interpretation

Transfer function  $c(sI - A)^{-1}b =: \frac{p(s)}{q(s)}$ , then  $r = \deg(q(s)) - \deg(p(s))$

Interpretation in time-domain:

## Theorem (Byrnes-Isidori form)

$(A, b, c)$  has relative degree  $r \in \{1, \dots, n\}$  if and only if there exists a coordinate transformation  $T$  such that  $\begin{pmatrix} \eta \\ z \end{pmatrix} = Tx$  such that  $y = \eta_1, \dot{y} = \eta_2, \dots, y^{(r-1)} = \eta_r$

$$y^{(r)} = a_{12} \begin{pmatrix} y \\ \vdots \\ y^{(r-1)} \end{pmatrix} + a_{22}z + \gamma u$$
$$\dot{z} = A_{21} \begin{pmatrix} y \\ \vdots \\ y^{(r-1)} \end{pmatrix} + A_{22}z$$

# Zero dynamics

$$\dot{x} = Ax + bu$$

$$y = cx$$

$$y^{(r)} = a_{12} \begin{pmatrix} y \\ \vdots \\ y^{(r-1)} \end{pmatrix} + a_{22}z + \gamma u$$

$$\dot{z} = A_{21} \begin{pmatrix} y \\ \vdots \\ y^{(r-1)} \end{pmatrix} + A_{22}z$$

## Question

Which input  $u$  is needed to keep output  $y$  identically zero?

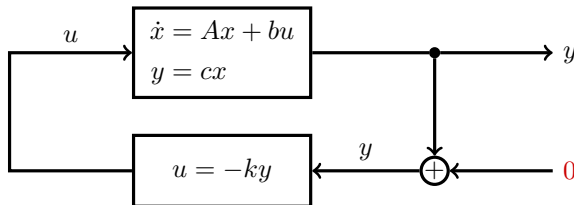
Byrnes-Isidori form for identically zero output:

$$0 = a_{22}z + \gamma u$$

$$\dot{z} = A_{22}z \quad \longleftarrow \text{zero dynamics}$$

**Answer:**  $u(t) = -\frac{1}{\gamma} a_{22} e^{A_{22}t} z(0) \rightarrow \infty$  if  $A_{22}$  has “bad” eigenvalues!

# High gain stabilization for r.d.-one systems



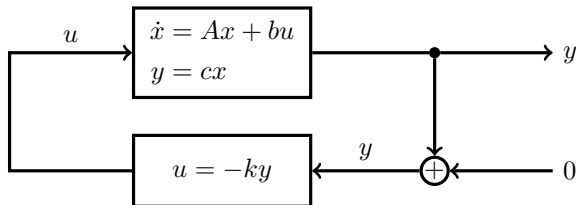
Assumptions:

- › **Relative degree  $r = 1$**   $\Leftrightarrow \gamma := cb \neq 0$ , in particular:

$$\begin{aligned} \text{System} \quad &\Leftrightarrow \quad \begin{aligned} \dot{y} &= a_{11}y + a_{12}z + \gamma u \\ \dot{z} &= a_{21}y + A_{22}z \end{aligned} \end{aligned}$$

- › **positive high frequency gain**  $\Leftrightarrow \gamma > 0$
- › **stable zero-dynamics (minimum phase)**  $\Leftrightarrow A_{22}$  Hurwitz

# High gain stabilization for r.d.-one systems



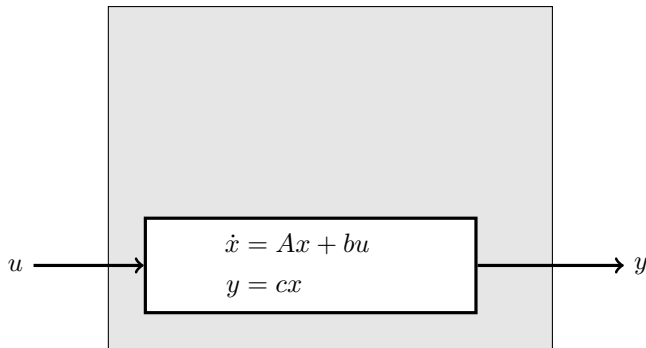
## Theorem (High-gain stabilization)

$cb > 0$  and stable zero-dynamics

$\Rightarrow \exists K > 0 \forall k \geq K$  : Closed loop is *asymptotically stable*

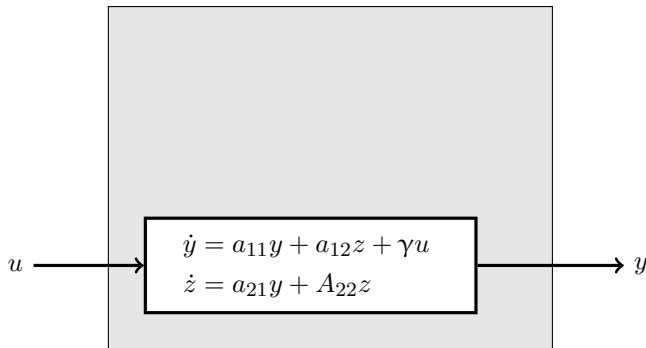
Key idea of proof: Show that  $\begin{bmatrix} a_{11} - \gamma k & a_{12} \\ a_{21} & A_{22} \end{bmatrix}$  is Hurwitz for sufficiently large  $k$ .

# From linear to nonlinear systems



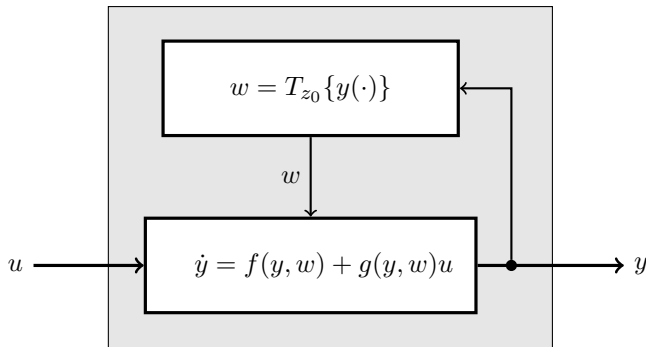


# From linear to nonlinear systems





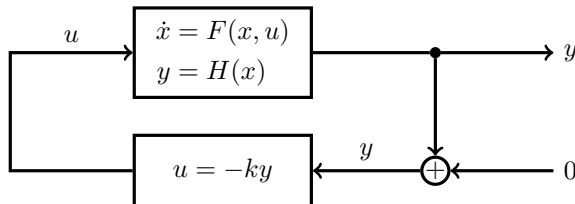
# From linear to nonlinear systems



Assumptions:

- ›  $T_{z_0}$  is **causal BIBO operator**, i.e.  $\exists \kappa(\cdot) : \|w\| \leq \kappa(\|y\|)$
- ›  $f$  and  $g$  continuous and  $g > 0$

# High gain stabilization for nonlinear systems



## Theorem

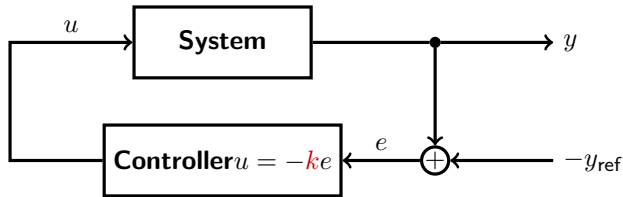
Assume there exists (nonlinear) coordinate transformation such that system is equivalent to

$$\dot{y} = f(y, w) + g(y, w)u, \quad w = T_{z_0}\{y(\cdot)\}$$

with  $f, g$  continuous,  $T_{z_0}$  causal BIBO operator and  $g > 0$ , then

$$\forall y(0) \forall z_0 \exists K > 0 \forall k \geq K : y(t) \rightarrow 0$$

# Summary high gain feedback



**Goal:** Output tracking

**Challenge:** Unknown system parameters

**Structural assumptions**

- › Relative degree one with known sign of “high frequency gain”
- › Stable zero dynamics

**High gain feedback:**  $u = -ke$  “works” for sufficiently large gain  $k > 0$

**Remaining challenge:** When is  $k$  sufficiently large?

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## Adaptive choice of gain

Adaptive stabilization

$\lambda$ -tracking

## The funnel controller

The original funnel controller with proof sketch

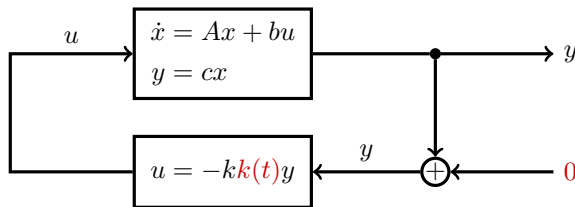
Relative degree two funnel controller

Bang-bang funnel control

Funnel synchronization

## Summary

# Choosing gain adaptively, linear case



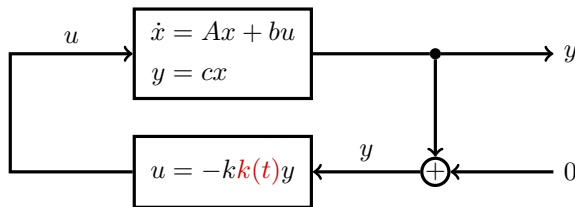
## Theorem (High-gain stabilization)

$cb > 0$  and stable zero-dynamics  $\Rightarrow \exists K > 0 \forall k \geq K : y(t) \rightarrow 0$

## Key idea

Why not make  $k$  time-varying with  $\dot{k}(t) > 0$  as long as  $y(t) > 0$ ?

# Choosing gain adaptively, linear case



Theorem (Adaptive High-Gain Feedback, BYRNES & WILLEMS 1984)

$cb > 0$  and stable zero-dynamics  $\Rightarrow$

$\dot{k}(t) = y(t)^2$  makes closed loop *asymptotically stable*

and  $k(\cdot)$  remains *bounded*

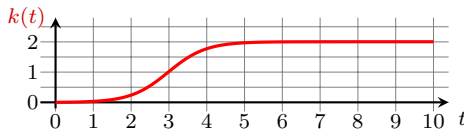
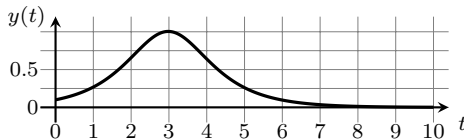
Boundedness of  $k(t) = \int_0^t y(s)^2 \, ds$  follows from final *exponential decay of  $y$* .



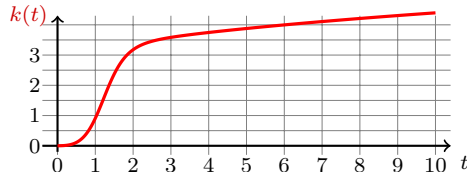
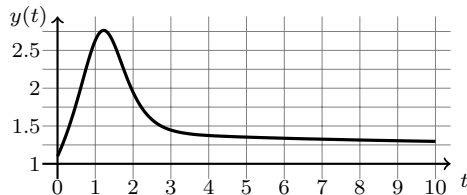
# Simulations

$$\dot{y} = y + u, \quad u(t) = -k(t)(y(t) - y_{\text{ref}}(t)), \quad \dot{k} = (y - y_{\text{ref}})^2$$

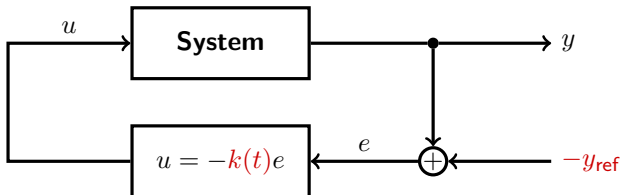
output and gain for  $y_{\text{ref}} = 0$



output and gain for  $y_{\text{ref}} = 1$



# High gain adaptive control and tracking?

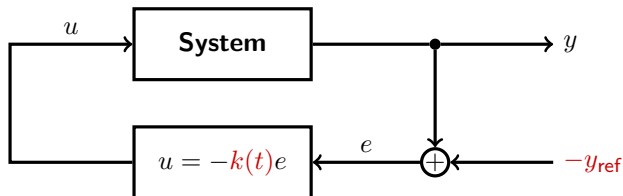


## Unbounded gain

For  $y_{\text{ref}} \neq 0$  the adaptation rule  $\dot{k} = e^2$  leads to unbounded gain.

Recall: Constant gain for  $y_{\text{ref}} \neq 0$  only leads to **practical tracking**, i.e.  $e(t) \not\rightarrow 0$

# High gain adaptive control and tracking?

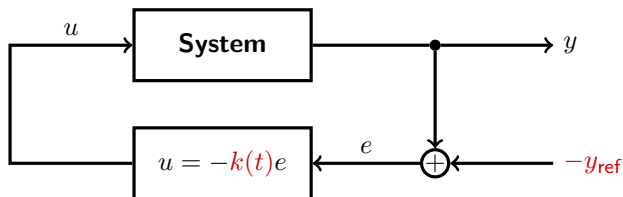


How to prevent unbounded growth?

Stop increasing gain when error is sufficiently small, e.g. via

$$\dot{k}(t) = \begin{cases} 0 & |e(t)| \leq \lambda \\ |e(t)|(|e(t)| - \lambda) & |e(t)| > \lambda \end{cases}$$

# High gain adaptive control and tracking?



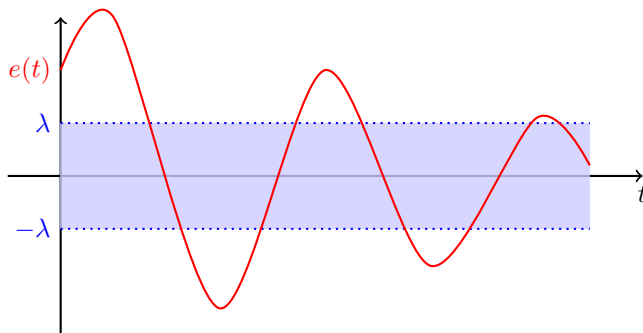
## Theorem ( $\lambda$ -tracking, ILCHMANN & RYAN 1994)

Assume r.d.-one with “ $\gamma > 0$ ”, stable zero-dynamics and  $y_{\text{ref}}, \dot{y}_{\text{ref}}$  *bounded*. For  $\lambda > 0$  consider

$$\dot{k}(t) = \begin{cases} 0, & |e(t)| \leq \lambda, \\ |e(t)|(|e(t)| - \lambda), & |e(t)| > \lambda. \end{cases}$$

Then the closed loop is *practically stable*, i.e.  $\limsup_{t \rightarrow \infty} |e(t)| \leq \lambda$ .

# Remaining problems of $\lambda$ -tracker



Problems:

- › No guarantees **when**  $|e(t)| \leq \lambda$
- › No bounds on **transient behaviour**
- › Monotonically **growing**  $k(\cdot)$   $\Rightarrow$  Measurement noise unnecessarily amplified

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The original funnel controller with proof sketch

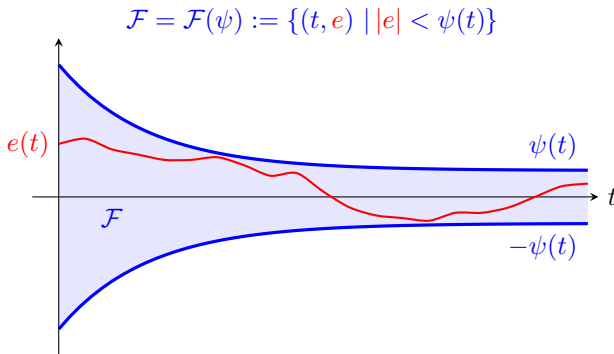
Relative degree two funnel controller

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Funnel synchronization

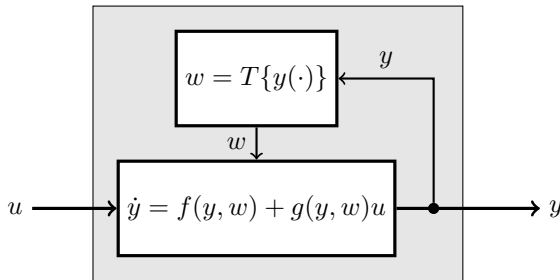
## Summary

# The funnel as time-varying error bound



Idea:  $k(t)$  **large**  $\iff$  Distance of  $e(t)$  to funnel boundary **small**  
 $\leadsto$  **Funnel gain:**  $k(t) = \frac{1}{\psi(t) - |e(t)|}$

# Funnel controller works



## System class

Equivalent to structure left:

- ›  $T$  is causal and BIBO
- ›  $f, g$  continuous
- ›  $g > 0$

## Theorem (ILCHMANN, RYAN, SANGWIN 2002)

Assume  $y_{\text{ref}}, \dot{y}_{\text{ref}}, \psi, \dot{\psi}$  **bounded**,  $\liminf_{t \rightarrow \infty} \psi(t) > 0$  and  $|e(0)| < \psi(0)$  where  $e := y - y_{\text{ref}}$ . Then

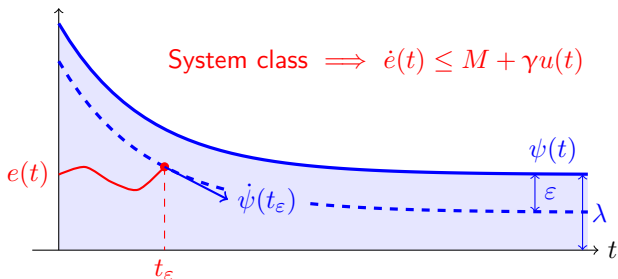
$$u(t) = -k(t)e(t) \quad \text{with} \quad k(t) = \frac{1}{\psi(t) - |e(t)|}$$

ensures that  $e(t)$  **remains within funnel  $\mathcal{F}(\psi)$**  while  $k(t)$  **remains bounded**.





## Step 2b: Funnel invariant (case $e(t) > 0$ )



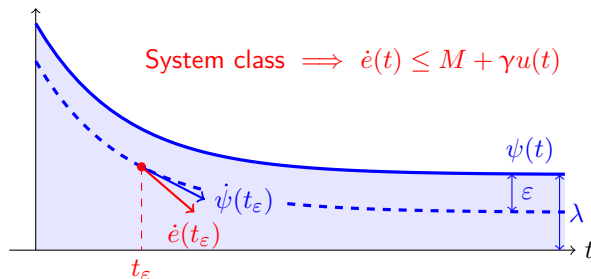
Assumptions:  $\varepsilon < \psi(0) - e(0)$        $\varepsilon < \lambda/2$        $\psi(t) \geq \lambda$

$$e(t_\varepsilon) = \psi(t_\varepsilon) - \varepsilon \quad \Rightarrow \quad k(t_\varepsilon) = \frac{1}{\psi(t_\varepsilon) - |e(t_\varepsilon)|} = \frac{1}{\varepsilon}$$

$$\Rightarrow \quad u(t_\varepsilon) = -k(t_\varepsilon)e(t_\varepsilon) \leq -\frac{1}{\varepsilon} \frac{\lambda}{2}$$

$$\Rightarrow \quad \dot{e}(t_\varepsilon) \leq M - \frac{\gamma\lambda}{2\varepsilon}$$

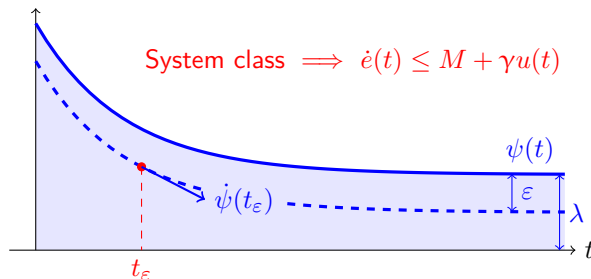
## Step 2b: Funnel invariant (case $e(t) > 0$ )



Assume  $\dot{\psi}(t) > -\Psi$  and  $\epsilon \leq \frac{\gamma\lambda}{2(\Psi + M)}$  we have

$$\dot{e}(t_\epsilon) \leq M - \frac{\gamma\lambda}{2\epsilon} \leq -\Psi < \dot{\psi}(t_\epsilon)$$

## Step 2b: Funnel invariant (case $e(t) > 0$ )



**Consequence:** For sufficiently small  $\varepsilon > 0$ ,

$$\mathcal{F}_\varepsilon := \{(t, e) \mid |e(t)| < \psi(t) - \varepsilon\}$$

is **positively invariant**, i.e.

$$(0, e(0)) \in \mathcal{F}_\varepsilon \implies (t, e(t)) \in \mathcal{F}_\varepsilon \quad \forall t \geq 0$$

and  $\omega < \infty$  **impossible!**

# Extensions of funnel controller

- › Asymptotic tracking (LEE & TRENN 2019)
- › Multi-Input Multi-Output (MIMO) (already in ILCHMANN ET AL. 2002)
- › Higher relative degree (ILCHMANN ET AL. 2007, BERGER ET AL. 2018)
- › Input saturation (ILCHMANN ET AL. 2004, HOPFE ET AL. 2010)
- › Bang-Bang funnel control (LIBERZON & TRENN 2013)
- › Funnel synchronization for multi-agent systems (SHIM & TRENN 2015)
- › For DAE-systems (BERGER 2016)

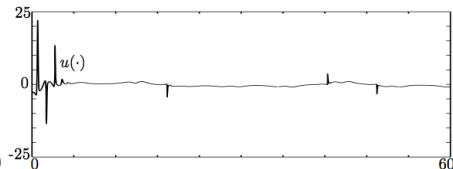
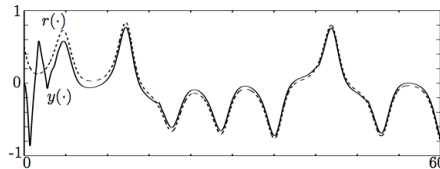
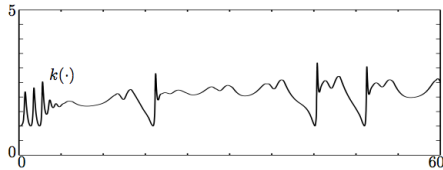
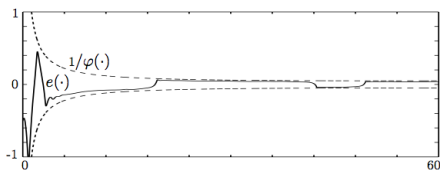
# Relative degree two via backstepping

For rel. deg. two systems, Funnel Controller is given by (ILCHMANN ET AL. 2007):

$$u(t) = -k(t)e(t) - (\|e(t)\|^2 + k(t)^2)k(t)^4(1 + \|\xi(t)\|^2)(\xi(t) + k(t)e(t))$$

$$k(t) = 1/(1 - \varphi(t)^2\|e(t)\|^2)$$

$$\dot{\xi}(t) = -\xi(t) + u(t)$$



Taken from: ILCHMANN, RYAN, TOWNSEND 2007, SICON

# Alternative Approach for relative degree two

Use **two** funnels, one for error and one for derivative of error

## Simple Control Law

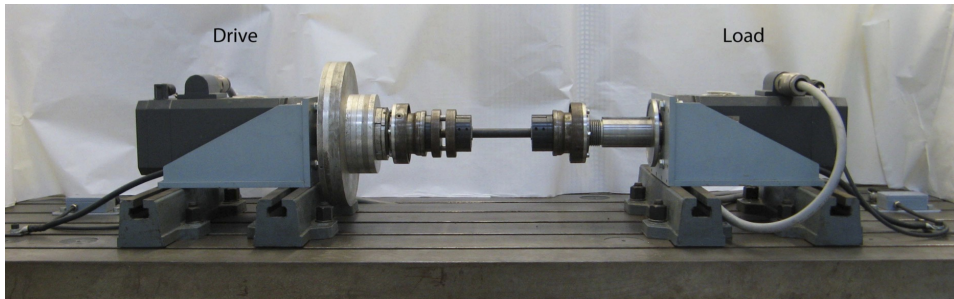
$$u(t) = -k_0(t)^2 e(t) - k_1(t) \dot{e}(t)$$
$$k_i(t) = \frac{1}{\psi_i(t) - |e(t)|}, \quad i = 0, 1$$

System class:  $\ddot{y}(t) = f(p_f(t), T_f\{y, \dot{y}\}(t)) + g(p_g(t), T_g\{y, \dot{y}\}(t))u(t)$

## Theorem (HACKL ET AL. 2012)

*The above Funnel Controller for relative-degree-two-systems works (under mild assumptions on  $\psi_0$  and  $\psi_1$ ).*

# Experimental verification



$$\begin{aligned}\dot{x}(t) &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \gamma \end{bmatrix} (u(t) + u_L(t) - (Tx_2)(t)), \\ y(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x(t),\end{aligned}$$

$x_1$ : angle of rotating machine

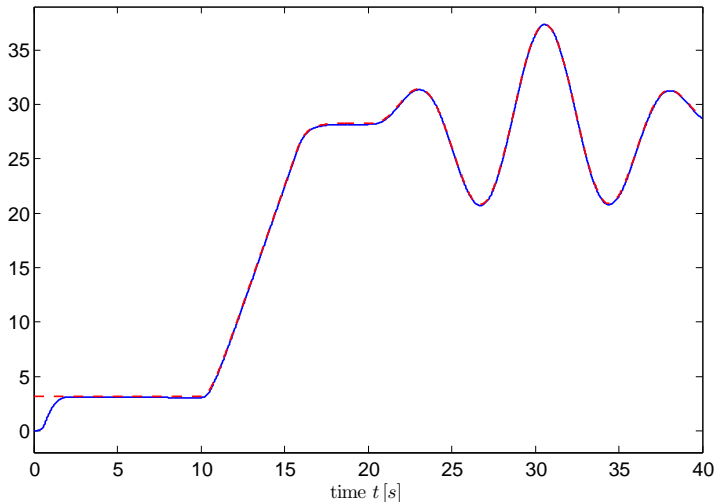
$x_2 = \dot{x}_1$ : angular velocity

$u_L$ : unknown load

$T: \mathcal{C}(\mathbb{R}_{\geq 0} \rightarrow \mathbb{R}) \rightarrow \mathcal{L}_{\text{loc}}^{\infty}(\mathbb{R}_{\geq 0} \rightarrow \mathbb{R})$  friction operator



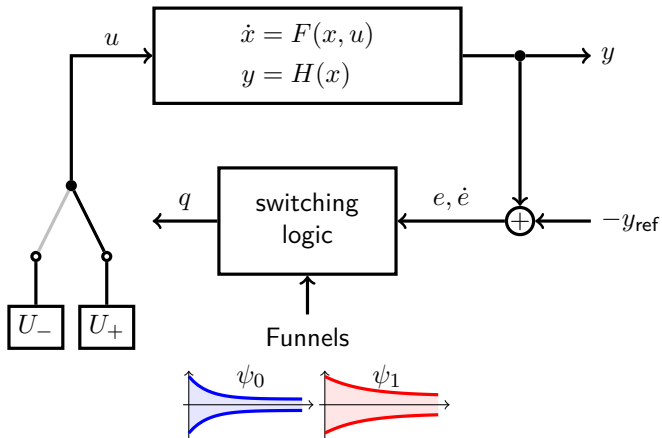
# Tracking control in experiment



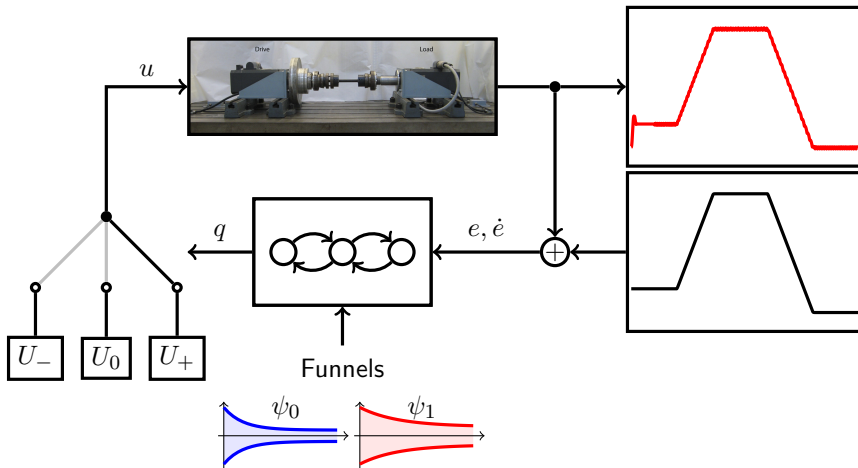
— Measured angle  $y(t)$  in rad, - - - reference angle  $y_{\text{ref}}(t)$  in rad



# Bang-Bang Funnel Control



# Bang-Bang Funnel Control



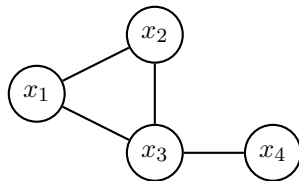
# Funnel synchronization - setup

## Given

- ›  $N$  agents with **individual**  $n$ -dimensional dynamics:

$$\dot{x}_i = f_i(t, x_i) + u_i$$

- › undirected connected coupling-graph  $G = (V, E)$
- › **local** feedback  $u_i = \gamma_i(x_i, x_{\mathcal{N}_i})$



## Desired

Control design for practical synchronization

$$x_1 \approx x_2 \approx \dots \approx x_n$$

$$u_1 = \gamma_1(x_1, x_2, x_3)$$

$$u_2 = \gamma_2(x_2, x_1, x_3)$$

$$u_3 = \gamma_3(x_3, x_1, x_2, x_4)$$

$$u_4 = \gamma_4(x_4, x_3)$$

# A „high-gain“ result

Let  $\mathcal{N}_i := \{j \in V \mid (j, i) \in E\}$  and  $d_i := |\mathcal{N}_i|$  and  $\mathcal{L}$  be the Laplacian of  $G$ .

## Diffusive coupling

$$u_i = -k \sum_{j \in \mathcal{N}_i} (x_i - x_j) \quad \text{or, equivalently,} \quad u = -k \mathcal{L} x$$

## Theorem (Practical synchronization, KIM et al. 2013)

*Assumptions:  $G$  connected, all solutions of **average dynamics***

$$\dot{s}(t) = \frac{1}{N} \sum_{i=1}^N f_i(t, s(t))$$

*remain **bounded**. Then  $\forall \varepsilon > 0 \exists K > 0 \forall k \geq K$ : Diffusive coupling results in*

$$\limsup_{t \rightarrow \infty} \|x_i(t) - x_j(t)\| < \varepsilon \quad \forall i, j \in V$$

# Remarks on high-gain result

## Common trajectory

It even holds that

$$\limsup_{t \rightarrow \infty} |x_i(t) - s(t)| < \varepsilon/2,$$

where  $s(\cdot)$  solves  $\dot{s}(t) = \frac{1}{N} \sum_{i=1}^N f_i(t, s(t)), \quad s(0) = \frac{1}{N} \sum_{i=1}^N x_i.$

Independent of coupling structure and amplification  $k$ .

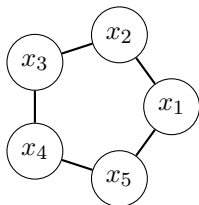
## Error feedback

With  $e_i := x_i - \bar{x}_i$  and  $\bar{x}_i := \frac{1}{d_i} \sum_{j \in \mathcal{N}_i} x_j$  diffusive coupling has the form

$$u_i(t) = -k e_i(t)$$

**Attention:**  $e_i \neq x_i - s$ , in particular, agents do not know „limit trajectory“  $s(\cdot)$

# Example (taken from KIM et al. 2015)



Simulations in the following for  $N = 5$  agents with dynamics

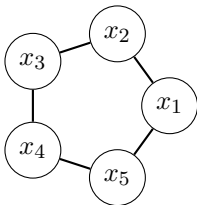
$$f_i(t, x_i) = (-1 + \delta_i)x_i + 10 \sin t + 10m_i^1 \sin(0.1t + \theta_i^1) + 10m_i^2 \sin(10t + \theta_i^2),$$

with randomly chosen parameters  $\delta_i, m_i^1, m_i^2 \in \mathbb{R}$  and  $\theta_i^1, \theta_i^2 \in [0, 2\pi]$ .

Parameters identical in all following simulations, in particular  $\delta_2 > 1$ , hence agent 2 has **unstable dynamics** (without coupling).



# Example (taken from KIM et al. 2015)

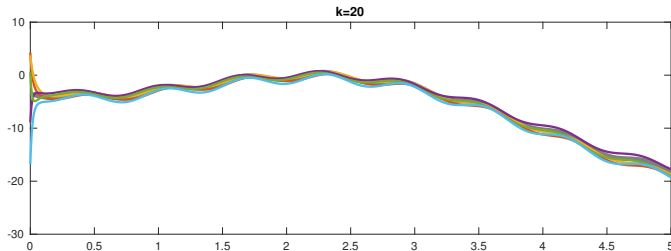
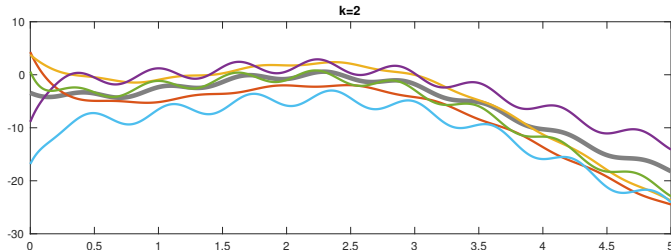


$$u = -k \mathcal{L} x$$

gray curve:

$$\dot{s}(t) = \frac{1}{N} \sum_{i=1}^N f_i(t, s(t))$$

$$s(0) = \frac{1}{N} \sum_{i=1}^N x_i(0)$$



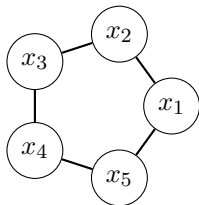
# Funnel synchronization: Initial idea

Reminder diffusive coupling:  $u_i = -k_i e_i$  with  $e_i = x_i - \bar{x}_i$ .

Combine diffusive coupling with Funnel Controller

$$u_i(t) = -k_i(t) e_i(t) \quad \text{with} \quad k_i(t) = \frac{1}{\psi(t) - |e_i(t)|}$$

# First simulations



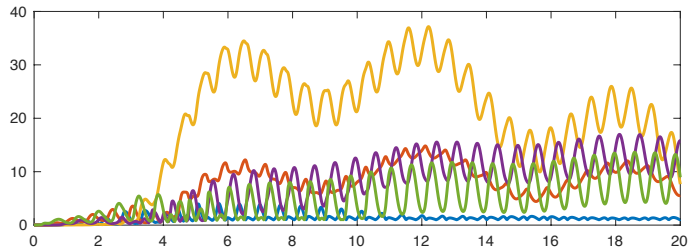
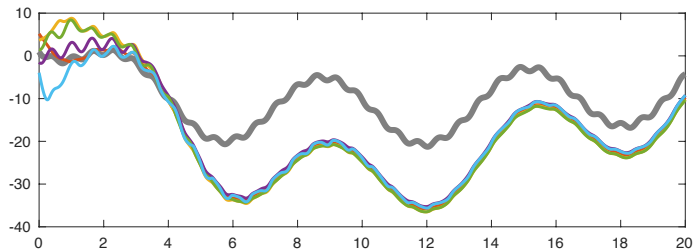
$$u_i(t) = -k_i(t)e_i(t)$$

$$k_i(t) = \frac{1}{\psi(t) - |e_i(t)|}$$

$$\psi(t) = \underline{\psi} + (\bar{\psi} - \underline{\psi})e^{-\lambda t}$$

$$\bar{\psi} = 20, \underline{\psi} = 1,$$

$$\lambda = 1$$



# Observations from simulations

## Funnel synchronization seems to work

- › errors remain within funnel
- › practical synchronizations is achieved
- › **limit trajectory** does **not** coincide with solution  $s(\cdot)$  of

$$\dot{s}(t) = \frac{1}{N} \sum_{i=1}^N f_i(t, s(t)), \quad s(0) = \frac{1}{N} \sum_{i=1}^N x_i(0).$$

## What determines the new limiting trajectory?

- › Coupling graph?
- › Funnel shape?
- › Gain function?

# Diffusive coupling revisited

## Diffusive coupling for weighted graph

$$u_i = -k \sum_i^N \alpha_{ij} \cdot (x_i - x_j) \quad \longrightarrow \quad u_i = - \sum_i^N k_{ij} \cdot \alpha_{ij} \cdot (x_i - x_j)$$

where  $\alpha_{ij} = \alpha_{ji} \in \{0, 1\}$  is the weight of edge  $(i, j)$

## Conjecture

If  $k_{ij} = k_{ji}$  are all sufficiently large, then practical synchronization occurs with desired limit trajectory  $s$  of **average dynamics**.

Proof technique from KIM et al. 2013 should still work in this setup.

# Edgewise Funnel synchronization

Diffusive coupling  $\rightarrow$  edgewise Funnel synchronization

$$u_i = - \sum_i^N k_{ij} \cdot \alpha_{ij} \cdot (x_i - x_j) \quad \longrightarrow \quad u_i = - \sum_i^N \textcolor{red}{k_{ij}(t)} \cdot \alpha_{ij} \cdot (x_i - x_j)$$

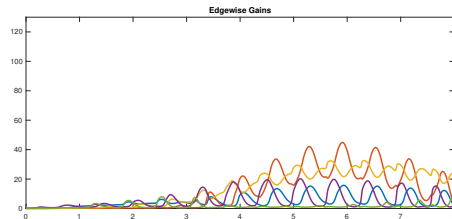
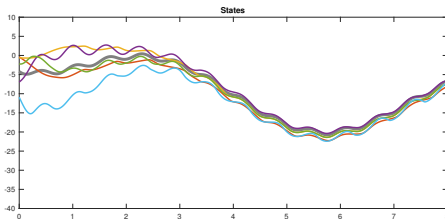
## Edgewise error feedback

$$k_{ij}(t) = \frac{1}{\psi(t) - |e_{ij}|}, \quad \text{with} \quad e_{ij} := x_i - x_j$$

Properties:

- › **Decentralized**, i.e.  $u_i$  only depends on state of neighbors
- › **Symmetry**,  $k_{ij} = k_{ji}$
- › **Laplacian feedback**,  $u = -\mathcal{L}_K(t, x)x$

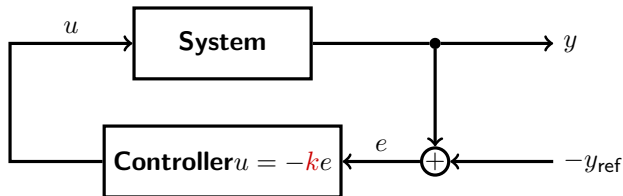
# Simulation (from TRENN 2017)



## Properties

- + Synchronization occurs
- + Predictable limit trajectory (given by average dynamics)
- + Local feedback law

# Summary high gain feedback and funnel control



**Goal:** Output tracking

**Challenge:** Unknown system parameters

**Structural assumptions**

- › Relative degree one with known sign of “high frequency gain”
- › Stable zero dynamics

**High gain feedback:**  $u = -ke$  “works” for sufficiently large gain  $k > 0$

**Funnel gain:**  $k(t) = \frac{1}{\psi(t) - |e(t)|}$  achieves tracking with prescribed performance