Observability and Determinability of Discrete Time Switched Linear Singular Systems: Multiple Switches Case

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1 Introduction

We consider the homogeneous discrete-time switched linear singular systems (SwLSS) of the form

$$E_{\sigma(k)}x(k+1) = A_{\sigma(k)}x(k), \tag{1a}$$

$$y(k) = C_{\sigma(k)}x(k), \quad k \in \mathbb{N} \tag{1b}$$

where $k \in \mathbb{N}$ is the time instant, $x(k) \in \mathbb{R}^n$ is the state, $y(k) \in \mathbb{R}^p, p \in \mathbb{N}$, is the output, $\sigma : \mathbb{N} \to \{0,1,2,...,\mathtt{p}\}$ is the switching signal determining which mode $\sigma(k)$ is active at time instant k, $E_i, A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $C_i \in \mathbb{R}^{p\times n}$, and $D_i \in \mathbb{R}^{p\times m}$ and E_i may be singular. We define the switching signal as follows

$$(\sigma_k), \quad \sigma(k) = \sigma_j \text{ if } k \in [k_j^s, k_{j+1}^s),\ j = \{0,1,2,...\} \tag{2}$$

where $k_j^s \in \mathbb{N}$ denote the switching times with $k_0^s = 0, k_{J+1}^s = K+1$, and $\sigma_j \in \{0,1,...,\mathtt{p}\}$. We assume here the switching signal is strictly increasing i.e. $k_{j+1}^s > k_j^s \ \forall j$. We study this system under index-1 assumption w.r.t. the given switching signal (2), see [1, 2] for the solution and the corresponding notations used in this study. The SwLSS (1) is called **observable** on $[0,K]$ w.r.t. the fixed switching signal (2) iff for all solutions on $[0,K]$ the following implication holds:

$$y^1 \equiv y^2 \Rightarrow x^1 \equiv x^2;$$

and the SwLSS (1) is called **determinable** on $[0,K]$ w.r.t. the fixed switching signal (2) iff the following implication holds:

$$y^1_{[0,K]} \equiv y^2_{[0,K]} \Rightarrow x^1(K) = x^2(K).$$

By $\mathbb{S}^{[\tau_D]}_{[0,K]}$ we define the set of all switching signals with each mode is active at least $\tau_D \in \mathbb{N}$ time steps.

2 Main Results

2.1 Observability

The main result for the observability characterization is presented in the following theorem.

Theorem 2.1 *Assume SwLSS* (1) *is of index-1 w.r.t. the fixed switching signal* (2). *It is observable on* $[0,K]$ *if, and only if,*

$$\mathscr{S}_{\sigma_0} \cap \bigcap_{j=0}^{J} [\Psi_\sigma(j,0)]^{-1}(\mathscr{O}_{\sigma_j}^{k_{j+1}^s - k_j^s - 1}) = \{0\} \tag{3}$$

where for every j,k $\mathscr{S}_0 = [A_0]^{-1}(\operatorname{im} E_0)$,

$$\Psi_\sigma(j,0) = \Phi_{\sigma_j,\sigma_{j-1}}\Phi_{\sigma_{j-1}}^{k_j^s - k_{j-1}^s - 1} \cdots \Phi_{\sigma_1,\sigma_0}\Phi_{\sigma_0}^{k_1^s - k_0^s - 1} I.$$

$$\mathscr{O}_{\sigma_j}^k = \ker[C_{\sigma_j}^\top, (C_{\sigma_j}\Phi_{\sigma_j})^\top, \dots, (C_{\sigma_j}\Phi_{\sigma_j}^k)^\top]^\top. \tag{4}$$

Note that $[*]^{-1}$ denotes the preimage and not the inverse. In general, the observability depends on the switching times as the switching times appear explicitly in the observability condition (3). We provide some situations in the following where the observability does not depend on the switching times i.e. **constant**.

Proposition 2.2 *Consider the two-dimensional SwLSS* (1) *of index-1 w.r.t. the switching signal* (2). *Then its observability is constant for every* $\sigma \in \mathbb{S}^{[2]}_{[0,K]}$ $\forall K \geq 4$.

Proposition 2.3 *Consider the SwLSS* (1) *of index w.r.t. the switching signal* (2). *If the unobservable subspace of the initial mode is invariant under* $\Phi_{\sigma(k),\sigma(k-1)}$ $\forall k \in \mathbb{N}$ *then its observability is constant for every* $\sigma \in \mathbb{S}^{[2]}_{[0,K]}$ $\forall K \geq 2n$.

2.2 Determinability

We define the following sequence for $j = 1,2,\dots,J$ and for $k \in (k_j^s, k_{j+1}^s - 1)$

$$\mathscr{Q}^0 = \mathscr{S}_0 \cap \ker C_0$$

$$\mathscr{Q}^k = \ker C_{\sigma_j} \cap A_{\sigma_j}\mathscr{Q}^{k-1}, k = k_j^s+1, k_j^s+2, \dots, k_{j+1}^s$$

$$\mathscr{Q}^{k_j^s} = \ker C_{\sigma_j} \cap A_{\sigma_{j-1}}\mathscr{Q}^{k_j^s-1}.$$

Theorem 2.4 *The SwLSS* (1) *is determinable on* $[0,K], K \in [k_J^s, k_{J+1}^s)$ *w.r.t. the fixed switching signal* (2) *if, and only if* $\mathscr{Q}^K = \{0\}$.

References

[1] Pham Ky Anh, et al. "The one-step-map for switched singular systems in discrete-time." *Proc. 58th IEEE Conf. Decision Control (CDC) 2019.*

[2] Sutrisno, Stephan Trenn. "Observability of Switched Linear Singular Systems in Discrete Time: Single Switch Case." *Proc. of European Control Conference (ECC). to appear.*