Model reduction of switched systems in time-varying approach

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1 Introduction

Consider switched linear systems of the form:

$$S_\sigma : \begin{cases} \dot{x}(t) = A_\sigma x(t) + B_\sigma u(t), \quad x(t_0) = \mathbf{0}, \\ y(t) = C_\sigma x(t), \end{cases} \tag{1}$$

where $\sigma : \mathbb{R} \to M = \{0, 1, 2, \cdots, s\}$ with finitely many switching times $t_0 < t_1 < t_2 < t_3 < \cdots < t_s$. The system matrices are $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $C_i \in \mathbb{R}^{p\times n}$, $D_i \in \mathbb{R}^{n\times m}$, where $i \in M$. Here, the i-th mode is active in the interval $[t_i, t_{i+1})$, for $i = 1, \cdots, s$.

Recently, many works have been done on model order reduction (MOR) of switched linear systems. We view here the switched system as a special *time-varying* system and aim for a (time-varying) model-reduction depending on a given (known) switching signal. A model reduction approach is proposed for piecewise constantly switched systems, based on balancing based MOR method for linear time-varying systems, for details see [1].

2 Problem setting

The available balanced truncation methods for time-varying system assume that the coefficient matrices are at least continuous. However, the switched system (1) seen as a time-varying system has *discontinuous* coefficient matrices, so we propose (continuously) time-varying approximation:

$$S_\varepsilon : \begin{cases} \dot{x}(t) = A_\varepsilon(t)x(t) + B_\varepsilon(t)u(t), \quad x(t_0) = \mathbf{0}, \\ y(t) = C_\varepsilon(t)x(t), \end{cases} \tag{2}$$

where, for example $A_\varepsilon(.)$ is defined as follows:

$$A_\varepsilon(t) = \begin{cases} A_i + \frac{t-t_i}{\varepsilon}(A_{i+1} - A_i) & : t \in [t_i, t_i + \varepsilon], \\ A_{i+1} & : t \in (t_i + \varepsilon, t_{i+1}] \end{cases}.$$

3 Model reduction

At first we compute the time-varying controllability and observability Gramians $P(t)$ and $Q(t)$ respectively. Let the time-varying system (2) is boundedly completely controllable and observable on $[t_0, t_f]$. Then there exists a time-varying coordinate transformation T such that

$$T(t)^{-1}P(t)T(t)^{-\top} = T(t)^\top Q(t)T(t) = \Pi(t),$$

for all $t \in [t_0, t_f]$. In fact, $T(t) = R(t)U(t)\Pi(t)^{-1/2}, T(t)^{-1} = \Pi(t)^{-1/2}V(t)^\top L(t)^\top$, where $U(t)\Pi(t)V(t)^\top = svd(R(t)^\top L(t))$, and where $R(t)R(t)^\top = P(t)$ and $L(t)L(t)^\top = Q(t)$ are the Cholesky decompositions of P and Q, respectively.

Using the projection matrices, we compute reduced system:

$$\hat{S}_\varepsilon : \begin{cases} \dot{\hat{x}}(t) = \hat{A}_\varepsilon(t)\hat{x}(t) + \hat{B}_\varepsilon(t)u(t), \quad \hat{x}(t_0) = \mathbf{0}, \\ \hat{y}(t) = \hat{C}_\varepsilon(t)\hat{x}(t), \end{cases} \tag{3}$$

where the system matrices are $\hat{A}_i \in \mathbb{R}^{r\times r}$, $\hat{B}_i \in \mathbb{R}^{r\times m}$, $\hat{C}_i \in \mathbb{R}^{p\times r}$, $D_i \in \mathbb{R}^{r\times m}$, for $i \in M$ and $r \ll n$.

4 Results

Consider a randomly generated SISO switched linear system

$$A_1 = \begin{bmatrix} -0.74 & 0.3 & 0.2 & -0.01 & -0.06 \\ 0.965 & -1.43 & -0.5 & 0.8 & -0.26 \\ 0.922 & -0.0487 & -0.44 & 0.03 & 0.054 \\ -0.98 & 0.28 & 0.31 & -0.764 & 0.07 \\ -0.634 & -1.26 & 0.534 & 0.662 & -0.48 \end{bmatrix}, B_1 = \begin{bmatrix} 2 \\ 1.4 \\ 1.1 \\ -0.06 \\ 0.08 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 2.5 & 2 & 1.6 & 0.02 & -0.03 \end{bmatrix}, A_2 = A_1 - 0.5 * I_5,$$

$$B_2 = \begin{bmatrix} 2.5 & 1.8 & .3 & 0.6 & -1 \end{bmatrix}^T, C_2 = \begin{bmatrix} 1.5 & 1.4 & .7 & 0.1 & 0.2 \end{bmatrix}, \varepsilon = 10^{-3}.$$

Consider switching times are $t_1 = 2s, t_2 = 4s$. Applying proposed technique, we compute first order reduced system. Figure 1 shows that the first order reduced system gives good approximation of original system.

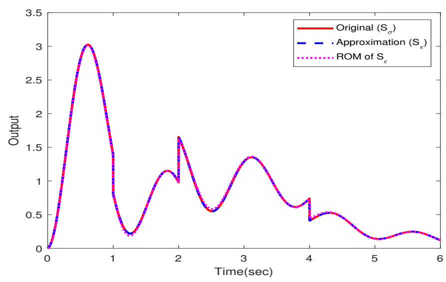

Figure 1: Comparison between the output of original system, proposed approximation and 1st order reduced system.

References

[1] N. Lang, J. Saak, T. Stykel "Balanced truncation model reduction for linear time-varying systems," Math. and Comp. Mod. of Dyn. Sys., 22, p. 267-281, 2016.