

**Registration form (basic details)****1a. Details of applicant**

Title: Dr.  
First Name: Stephan  
Surname: Trenn  
...

**1b. Title of research proposal**

Analysis and Control of Switched Differential Algebraic Equations

For short in the following: SWITCHEDDAEs

**1c. Summary of research proposal**

The combined presence of sudden structural changes and constrained dynamics in mathematical models of dynamical systems leads to non-existence of classical solutions. This problem occurs e.g. in models of power grids, electrical circuits, multibody systems or water distribution networks. Switched differential algebraic equations (switched DAEs) are a novel modeling framework for these dynamical systems. So far, switched DAEs are not used for modeling because neither a general solution theory nor control-theoretical methods are available. However, many systems need to be modeled as switched DAEs to capture essential effects like jumps or even Dirac impulses; the latter occur in reality e.g. in the form of sparks in electrical circuits or as water hammers in water networks.

In this VIDI project a distributional solution theory for nonlinear switched DAEs encompassing jumps and Dirac impulses will be developed. Based on the rigorous treatment of these impulsive effects, new diagnostic methods (e.g. observers and fault detectors) as well as new controller designs (in particular optimal controllers) will be derived. The distributional solution framework with its corresponding novel control theoretic approaches will not only be a mathematical breakthrough but will also have the potential to lead to sophisticated new methods to solve real world problems.

A special emphasis will be on analyzing models of the electrical power grid, which consist of the so called swing equations (ordinary differential equations) together with the power balance equations (nonlinear algebraic constraints). Faults or scheduled activation/deactivation of generators yield sudden structural changes of the power network (switches). The groundbreaking new diagnostic and control tools for switched DAEs will therefore have the potential to solve problems like the very pressing need to stabilize the power grid in the presence of an increasing number of renewable energy sources in order to prevent blackouts.

**1d. Keywords**

differential-algebraic equations; switched systems; solution theory; stabilization; optimal control

**1e. Current institution of employment**

TU Kaiserslautern, Germany

### 1f. Prospective host institution

University of Groningen, Johann Bernoulli Institut for Mathematics and Computer Science

### 1g. NWO Division

EW: Physical Sciences

### 1h. Main field of research

Code	Main field of research
11.90.00	Mathematics, other: Systems and Control
	<b>Other fields of research</b>
11.30.00	Functions, differential equations
16.20.00	Software, algorithms, control systems
14.30.00	Electrical engineering

### 1i. Public summary of your research proposal

Disconnecting power lines leads to large sparks (arc of light) at the circuit breakers. Occurrences of these sparks at the wrong place or at the wrong time can lead to disastrous effects. The novel mathematical modeling framework of switched differential algebraic equations allows to describe, analyze and prevent these sparks.

Het loskoppelen van hoogspanningslijnen leidt tot grote vonken (lichtbogen) op de stroomonderbrekers. Het optreden van dergelijke vonken op verkeerde tijdstippen en plaatsen kan desastreuze gevolgen hebben. Door hoogspanningsnetwerken te modelleren met de nieuwe modelklasse van geschakelde differentiaal-algebraïsche vergelijkingen kan het optreden van dergelijke vonken worden beschreven, geanalyseerd en voorkomen.

Research proposal

## 2a. Description of the proposed research

### 2a.1 Overall aim and key objectives

#### Introduction

Mathematical models play a key role for understanding, analyzing, monitoring and controlling machines, distribution networks or production processes. These models usually consists of differential equations (like Newton's law in mechanical systems) as well as algebraic equations (e.g. Kirchhoff's laws in electrical circuits). The overall model is then given by an implicit differential equation or, in other words, differential-algebraic equation (DAE). It is often possible (especially in linear models) to explicitly solve the algebraic constraints and to reformulate the model as an ordinary differential equation (ODE); this reformulation involves a change and elimination of variables. In certain applications sudden structural changes may occur, for example disconnecting power lines in power grids or breaking of joints in mechanical multibody systems. These structural changes lead to models with different algebraic constraints. In particular, the reduction to ODEs results in different variables, i.e. the resulting models are not compatible anymore. To analyze and control the effects of sudden structural changes it is therefore necessary to work with the original DAE model. Sudden structural changes in dynamical systems can be formulated in the framework of switched systems, which has been focused on ODEs so far; in order to capture the effects of changing algebraic constraints it is therefore necessary to consider the novel approach of modeling dynamical systems with switched DAEs.

The overall aim of SWITCHEDDAES is the developing of a pioneering solution framework and innovative control-theoretical tools for systems modeled by switched DAEs. Besides the potential for a breakthrough in control methods in real-world applications, SWITCHEDDAES will establish a new research area within the field of mathematical systems theory.

A major motivating example where modeling with switched DAEs is necessary to capture effects induced by sudden structural changes is the power grid. Faults or activation/deactivation of generators yield structural changes, i.e. a switched system. Each individual mode is modeled by the so-called swing equation (an ordinary differential equation describing the generator dynamics) together with the nonlinear power balance equation, resulting in a nonlinear DAE. The theoretical results of SWITCHEDDAES will therefore also be a breakthrough for the analysis and control of power grids.

#### From LTIs to switched DAEs

Historically, the main focus of mathematical control theory was on linear time-invariant (LTI) systems of the form

$$\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t) + Bu(t), \quad t \in \mathbb{R},$$

where  $x : \mathbb{R} \rightarrow \mathbb{R}^n$ ,  $n \in \mathbb{N}$ , is the state,  $u : \mathbb{R} \rightarrow \mathbb{R}^p$ ,  $p \in \mathbb{N}$ , is the input,  $y : \mathbb{R} \rightarrow \mathbb{R}^q$ ,  $q \in \mathbb{N}$ , is the output and  $A, B, C, D$  are matrices of appropriate size. However, the real world is not so simple and generalizations needed to be studied. LTIs can be generalized in three independent directions:

1. Dropping the assumption on linearity results in **nonlinear systems** of the form

$$\dot{x}(t) = f(x(t), u(t)), \quad y(t) = h(x(t), u(t)), \quad t \in \mathbb{R}$$

where  $f : \mathbb{R}^n \times \mathbb{R}^p \rightarrow \mathbb{R}^n$  and  $h : \mathbb{R}^n \times \mathbb{R}^p \rightarrow \mathbb{R}^q$ .

2. Including algebraic constraints results in **differential algebraic equations** of the form

$$E\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t) + Du(t), \quad t \in \mathbb{R}$$

where  $E$  is a singular (possibly rectangular) matrix.

3. Allowing (piecewise constant) time-variance results in **switched systems** of the form

$$\dot{x}(t) = A_k x(t) + B_k u(t), \quad y(t) = C_k x(t) + D_k u(t), \quad t \in [t_k, t_{k+1}),$$

where  $k \in \mathbb{N}$  is the current mode of the system and  $t_k \in \mathbb{R}$  is the corresponding switching instant. Introducing the switching signal  $\sigma : \mathbb{R} \rightarrow \mathbb{N}$ , a switched system can compactly be written as

$$\dot{x}(t) = A_{\sigma(t)} x(t) + B_{\sigma(t)} u(t), \quad y(t) = C_{\sigma(t)} x(t) + D_{\sigma(t)} u(t), \quad t \in \mathbb{R}.$$

These extensions are each for themselves very well motivated and state-of-the-art in systems theory. The same is true for nonlinear DAEs and nonlinear switched systems. Since the underlying solution framework (absolutely continuous functions) remained unchanged for these generalizations the results were more or less straightforward combinations of the results of each of the individual extensions.

The picture **drastically changes** when considering switched DAEs of the form (linear)

$$E_\sigma \dot{x} = A_\sigma x + B_\sigma u, \quad y = C_\sigma x + D_\sigma u,$$

or (nonlinear)

$$\boxed{E_\sigma \dot{x} = f_\sigma(x, u), \quad y = h_\sigma(x, u).} \quad (1)$$

**Classical solutions cannot be expected anymore** because structural changes lead to the problem that the algebraic constraints are usually not fulfilled anymore after a switch. These violations of the algebraic constraints induce **jumps** in the state variable  $x$  and are a distinguishing feature of switched DAEs. Furthermore, jumps in the state can even produce **Dirac impulses** in the solution. Hence the solution theory of (1) needs to consider solutions which are distributions (generalized functions in the sense of [Schwartz 1957, 1959]). For the linear case the PI already solved this fundamental problem in his PhD thesis [Trenn 2009], but the combination of all three extensions in the form (1) pose a mathematical challenge because a nonlinear evaluation of a distribution is not defined.

### Objectives and State-of-the-Art

The project SWITCHEDDAEs aims at developing a solution theory for (1) and the development of innovative control-theoretical methods, in particular, observer design and mode detectors, controller design for stabilization and the design of optimal controllers. Furthermore, the theoretical results will be verified against real-world problems. The corresponding five work packages are as follows:

- Objective 1: Distributional solution theory
- Objective 2: Diagnosis of switched DAEs
- Objective 3: Stabilization of switched DAEs
- Objective 4: Optimal Control
- Objective 5: Application to models of real world problems

**Objective 1: Distributional solution theory**

It is essential to consider a non-standard solution framework for switched DAE; in particular, the restriction on absolutely continuous solution as it is common for switched ODEs [Liberzon 2003] is not feasible. This is a very crucial property of switched DAEs and can already be observed when considering very trivial examples, like the following electric circuit:

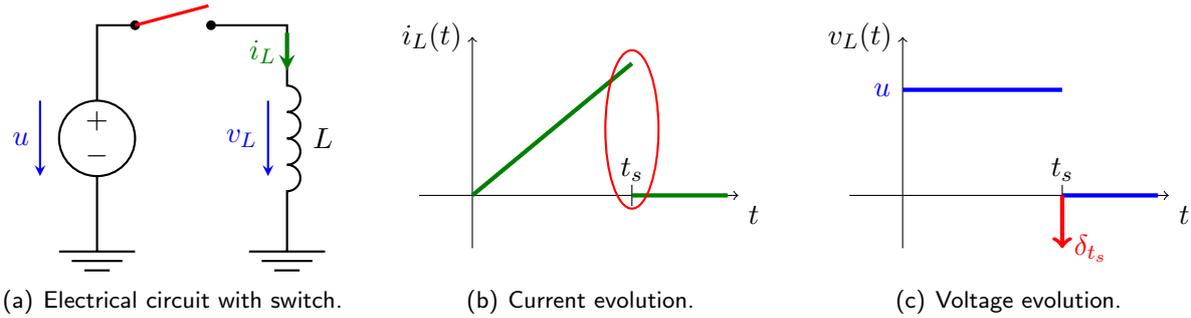


Figure 1: Electrical circuit with switch and corresponding solution.

**EXAMPLE: SIMPLE ELECTRICAL CIRCUIT.** Consider an electrical circuit as shown in Figure 1(a) consisting of an ideal voltage source in series with a switch and an inductor. The physical equations consists of the inductivity law for an ideal coil given by  $v_L = L \frac{d}{dt} i_L$ , the Kirchhoff law  $u = v_L$  if the switch is closed and  $i_L = 0$  if the switch is open. This results in the following two DAE models where  $x = [i_L, v_L]^T$ :

$$\text{switch closed: } \begin{bmatrix} L & 0 \\ 0 & 0 \end{bmatrix} \dot{x} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u, \quad \text{switch open: } \begin{bmatrix} L & 0 \\ 0 & 0 \end{bmatrix} \dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ -1 \end{bmatrix} u.$$

Assume that the switch is closed initially and opened after some time  $t_s > 0$ . For a constant non-zero input voltage  $u$  the corresponding solutions for  $i_L$  and  $v_L$  are plotted in Figures 1(b) and 1(c). Due to the inductivity law the current grows linearly as long as the switch is closed (Figure 1(b)). After the switch is opened the current is zero (and the inductivity law also yields  $v_L(t) = 0$  for  $t > t_s$ ), in particular there is a jump in the current  $i_L$  at time  $t = t_s$ . However, the inductivity law is valid also *at the switching instant*, hence the voltage must contain the (distributional) derivative of a jump, i.e. a *Dirac impulse*  $\delta_{t_s}$  as shown in Figure 1(c). This Dirac impulse also occurs in reality in the form of a voltage spark and is therefore *not just a mathematical artifact*.  $\triangleleft$

As switched nonlinear DAEs of the general form (1) have not yet been studied so far, a necessary first step in SWITCHEDDAES is the development of a suitable solution theory; in particular, the induced jumps (due to the changing algebraic constraints) as well as the presence of Dirac impulses (due to a possible higher index of the DAEs) shall be part of the solution theory.

A solution theory for *linear* switched DAEs of the form

$$\begin{aligned} E_\sigma \dot{x} &= A_\sigma x + B_\sigma u, \\ y &= C_\sigma x + D_\sigma u, \end{aligned} \tag{2}$$

where  $E_k, A_k, B_k, C_k, D_k$ ,  $k \in \{1, 2, \dots, P\}$  are matrices of suitable size, was studied by the PI in his PhD-thesis [Trenn 2009] (see also the book chapter [Trenn 2012]). In order to handle jumps and Dirac impulses it is necessary to consider distributional solutions. This approach cannot be easily extended to the nonlinear case because if  $x$  contains Dirac impulses, then it is not clear how a nonlinear function of  $x$  shall be defined (e.g. what is the sine of the Dirac impulse?). The PI

circumvented this problem in [Liberzon and Trenn 2012] by excluding Dirac impulses in the solutions of (1) by assumption; however, as Dirac impulses in the solution correspond to real observable physical effects (e.g. sparks in electrical circuits) this assumption is too restrictive and it remains a challenge to formulate a suitable solution definition which also allows for a physical interpretation.

Other researches have been studying similar problems and their results (and the remaining challenges) are summarized in the following. In the PhD thesis [Wunderlich 2008] (and the subsequent journal publication [Mehrman and Wunderlich 2009]) a systems class similar to (1) was studied. There, in contrast to (1), the jumps at the switching instants are given explicitly and solutions are defined piecewise (determined by the active DAE between the switches and the jumps at the switches). In particular, this framework is not suitable for studying induced jumps and Dirac impulses. The solution theory of (1) is strongly connected with the problem of inconsistent initial values for non-switched DAEs, which is a long standing problem in electrical engineering (see e.g. [Frasca et al. 2010; Opal and Vlach 1990; Verghese, Levy, and Kailath 1981]), however these results are restricted to the linear case. Inconsistent initial values for nonlinear DAEs are usually only studied from a numerical viewpoint (see e.g. the recent monographs [Lamour, März, and Tischendorf 2013; Simeon 2013]) and the question is how to find a consistent initial value in a numerically stable way; in particular, the induced Dirac impulses in response to an inconsistent initial value are not studied so far.

A solution theory for (1) encompasses of course a solution theory for non-switched DAEs, which by itself is a challenging research topic. The results already available in the literature, e.g. [Rabier and Rheinboldt 1994; Reich 1990; Schlacher and Zehetleitner 2004], will be exploited and the focus will be on the effects of the switches. In case of a higher index, discontinuities in the input may lead to additional Dirac impulses even at non-switching times, hence a distributional solution framework will make a significant contribution also for non-switched DAEs.

The considered nonlinear switched DAE (1) contains as a special case also systems with state-dependent switching, for example systems of the form:

$$\begin{aligned} E_{\sigma(x)}\dot{x} &= A_{\sigma(x)}x + B_{\sigma(x)}u, \\ y &= C_{\sigma(x)}x + D_{\sigma(x)}u, \end{aligned} \quad (3)$$

where as above  $E_k, A_k, B_k, C_k, D_k, k \in \{1, 2, \dots, P\}$ , are matrices of suitable size, but now the switching signal  $\sigma$  is a function of the state  $x$  and not of the time. Systems of the form (3) are piecewise-linear systems and share many properties with (2). In particular, Dirac impulses and jumps are expected in the solutions of (3), but it is not completely understood in general how  $\sigma(x)$  has to be evaluated when  $x$  contains Dirac impulses. Some results are obtained within the powerful framework of complementarity systems, see e.g. [Çamlıbel et al. 2003] which also consider distributional solutions. It is one of the goals of SWITCHEDDAEs to relate the general distributional solution theory of (1) with the one used in the complementarity framework, which may lead to new insights for both modeling approaches.

## Objective 2: Diagnosis of switched DAEs

Even if a detailed system model of the form (1) is known it is usually not possible to exactly determine the current state  $x$ , because the initial conditions are not known in practice. However, for monitoring or stabilization it is often necessary to know  $x$  at least approximatively, but only the input  $u$ , the switching signal  $\sigma$  and the output  $y$  is known or measurable. The corresponding mathematical question is related to properties of the map  $x(\cdot) \mapsto y(\cdot)$  from the state to the output (for given switching signal  $\sigma$  and input  $u$ ) and a system is called observable when this map is injective. *A first aim of Objective 2 is therefore finding an observability characterization for switched DAEs.* For the

linear case this was already achieved by the PI in [Tanwani and Trenn 2012]; however, the nonlinear case is still unclear. In particular, the impulsive effects play a prominent role in the linear case and hence the general nonlinear results are expected to rely on the distributional solution theory from Objective 1.

Once observability is understood, *the next aim of Objective 2 is the design of an observer*, which is a dynamical system run in parallel to the system of interest and which produces an estimation of the state  $x$  given the input and output. For the linear case an observer is presented by the PI in [Tanwani and Trenn 2016], whose design and proof techniques strongly depend on linearity of the underlying model. The overall structure of the observer design may also be suitable for the nonlinear case, but the technical details of the generalization to the nonlinear case pose a mathematical challenge.

Finally, it cannot be assumed in general that the switching signal  $\sigma$  is known (e.g. when it is induced by faults), hence another “observability” question is concerned with the ability to determine the current mode of the system from the external signals. This problem is currently being investigated by the PI for the linear case, but for the nonlinear case no results are available, hence *designing a mode detector is another aim of Objective 2*.

### Objective 3: Stabilization of switched DAEs

Stability is an extremely important system property (instability in a real world system usually means that it blows up or breaks) and stabilization is one of the key challenges for all control applications. Stability for switched DAEs (1) within a simplified nonlinear solution framework was studied by the PI in [Liberzon and Trenn 2012]. Stabilization via the design of a suitable controller was not studied so far. For switched systems stabilization can be achieved via several approaches.

If the switching signal cannot be influenced, but is known, then it may be possible to find a feedback law  $u = F_\sigma(x)$  which renders the closed loop asymptotically stable. It is the *first aim of Objective 3 to find such a stabilizing feedback law for the linear case*.

Usually, the full state  $x$  is not known and the observer from Objective 2 has to be utilized; furthermore, the switching signal may not be known as well and has to be estimated with a mode detector from Objective 2. Combining a state feedback law with an observer and/or a mode detector may lead to undesired side effects; *the next aim of Objective 3 is therefore the analysis of these side effects*.

Instead of using a mode depending feedback, one may also use a mode-independent controller which just assumes some structural assumptions. A promising candidate controller for this purpose is the so called funnel controller. Funnel control was introduced in [Ilchmann, Ryan, and Sangwin 2002] for nonlinear system of relative degree one. Certain extensions have been developed by the PI, e.g. the consideration of input constraints for chemical reactor models [Ilchmann and Trenn 2004], the consideration of relative degree two cases [Hackl et al. 2013] and bang-bang funnel control for arbitrary relative degree [Liberzon and Trenn 2013]. There are some theoretical results concerning funnel control for linear DAEs [Berger, Ilchmann, and Reis 2012]; however, the nonlinear case or switches are not considered yet. *Another aim of Objective 3 is the generalization of the funnel controller to switched DAEs*.

If the switching signal can be influenced, then it may be possible to stabilize a system by appropriate switching resulting in a state-dependent switching law  $\sigma(t) = \sigma(x(t))$ . *The design of such a stabilizing switching law is a further aim of Objective 3*.

### Objective 4: Optimal control

The stabilizing controllers from Objective 3 do not take into account the “cost” of the control action, however in practice this is often the major deciding factor which controller will be really implemented. This leads to the question of finding an optimal controller. There are no results available

for optimal control of switched DAEs (even in the linear case), however the PI has recently obtained a duality result [Küstters and Trenn 2016] which may be utilized by formulating optimality conditions. *The aim of Objective 4 is therefore the design of an optimal controller* first for linear switched DAEs and later also for the nonlinear case.

#### Objective 5: Application to models of real world problems

The theoretical results, in particular solution theory and stabilization, will be applied to several examples from real world problems like power grids. For details see the section on knowledge utilization.

#### 2a.2 Research plan

The development of a distributional solution theory for nonlinear switched DAEs (1) as detailed in Objective 1 is crucial for the success of SWITCHEDDAEs because all other objectives rely at least partly on this solution theory; however even an “incomplete” solution theory will lead to breakthroughs in the other objectives. With the successful introduction of a mathematically consistent distributional solution framework for *linear* switched DAEs in [Trenn 2009] the PI proved capable of utilizing a sound mathematical understanding of distribution theory in the context of switched DAEs. The PI also introduced a simplified solution space (excluding Dirac impulses by assumption) for nonlinear switched DAEs in [Liberzon and Trenn 2012] which will be a starting point for the desired nonlinear results in Objectives 2, 3 and 4. There are different promising approaches to obtain a nonlinear distributional solution theory. One approach is to consider specially structured models where Dirac impulses occur in the state variables but are not part of the nonlinearities (these structures occur e.g. in models of water networks). Another approach is based on the observation that it is possible to assign a meaningful nonlinear evaluation of a Dirac impulse if the nonlinearity is bounded, which then covers general nonlinear terms which can be written as a sum of a (unbounded) linear term and a bounded nonlinear term. Finally, it is possible to consider an even larger space of generalized functions, e.g. the space of ultra functions introduced recently in [Benci and Baglini 2014] which allows nonlinear evaluations.

For Objective 2 the already known linear results concerning observability and observer design (obtained by the PI) will be a guide for the development of the general nonlinear results. Objectives 3 and 4 will first be considered in the linear case and afterwards generalizations to the nonlinear case will be developed. Objective 5 will be achieved by numerical simulations of realistic examples together with the verification of the feasibility of the assumptions formulated in the theoretical results of SWITCHEDDAEs.

Due to his experience in distributional solution theory, observer design, and stability of (linear) switched DAEs the PI will be well equipped to conduct the research concerning Objective 1, Objective 2 and Objective 3 by himself. The topic of optimal control for switched DAEs (Objective 4) is rather self-contained and is therefore a suitable project for a PhD student. Towards the end of the project, the PI and the PhD-student will be joined by an experienced researcher (i.e. a postdoc), ideally with some engineering background, who will investigate the application of the theoretical results obtained in Objectives 1-4. The corresponding time schedule is illustrated in Figure 2.

The PI is in close contact with experts from the three major fields (nonlinear systems theory, DAEs, switched systems) relevant for the success of SWITCHEDDAEs. It is planned to invite these experts for short visits (one to two weeks) in order to benefit from their ideas and experience. In particular, the PI is in close contact with the following internationally recognized experts:

- *Daniel Liberzon* (University of Illinois, USA) is one of the leading experts in switched systems and has collaborated with the PI in several journal and conference articles about stability of linear and nonlinear switched DAEs as well as nonlinear controller design.
- *Aneel Tanwani* (LAAS-CNRS, Toulouse, France) is an expert on observer design for switched

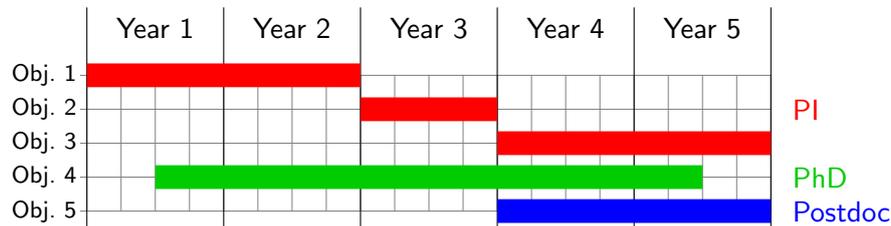


Figure 2: Time schedule for SWITCHEDDAEs

systems and has collaborated with the PI in several papers about observability and observer design for linear switched DAEs.

- *Francesco Vasca* (University of Sannio, Italy) is an expert in engineering applications of switched systems and has collaborated with the PI in several papers about averaging of linear switched DAEs.
- *Mohamed Djemai* (University of Valenciennes, France) is an expert in nonlinear observer design and invited the PI as a guest professor (one month) to Valenciennes in 2013.
- *Hyungbo Shim* (University of Seoul, South Korea) is a leading expert in the design of nonlinear observers and the PI is working with him on funnel control for synchronizing heterogeneous agents.

The PI is also in close contact with colleagues at the proposed host university, in particular, with most of the members of the Jan C. Willems Center for Systems and Control. He has worked together with Kanat Camlibel since 2011 on the connection between switched DAEs and the linear complementarity framework; recently, they have extended their collaboration towards the solution theory of DAEs with nonlinear constraints in the form of maximal monotone operators [Camlibel et al. 2016]. During his several visits to the University of Groningen, the PI also had discussions about switched DAEs and its relation to port Hamiltonian systems with Arjan van der Schaft. Recently, the PI has started a collaboration with Pietro Tesi on the topic of mode detection for switched systems. Finally, the PI had the pleasure to collaborate with the late Jan Willems [Trenn and Willems 2012] whose influence can still be felt strongly in Groningen. During SWITCHEDDAEs the PI will also strengthen his contacts with Claudio De Persis, Jaqueliën Scherpen and Bayu Jayawardhana, in particular with respect to Objective 5. Altogether, the Jan C. Willems Center for Systems and Control at the University of Groningen provides a perfect research environment for carrying out SWITCHEDDAEs and numerous fruitful collaborations are expected.

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## 2b. Knowledge utilization

### Potential

Mathematical control theory by nature is interdisciplinary, so all control theoretical results obtained in SWITCHEDDAEs have a potential utilization in many other disciplines. This includes electrical, mechanical and civil engineering, biology, chemistry and economy. The novel distributional solution theory for discontinuous implicit differential equations also has the potential for utilization in other mathematical areas like functional analysis and numerics.

The most significant potential for knowledge utilization of SWITCHEDDAEs is the application of the theoretical results to the modeling, analysis, stabilization and control of power grids. There are many different modeling approaches for the dynamics of power grids, however all models share the following two distinctive features: The models are subject to sudden structural changes due to topological changes in the power grid network (e.g. disconnecting of power lines, deactivation/activation of generators) and the dynamics are subject to algebraic constraints (power flow balances). These algebraic constraints are nonlinear and depend on the active topology. Hence the switched DAE model framework is the canonical starting point for model based analysis and control of power grids. However, the mathematical description of switched DAEs is not used so far, because of the lack of mathematical tools to deal with this description.

The results from Objective 1 can be utilized to analyze power grid models with respect to the effect of sudden structural changes. In particular, induced jumps and Dirac impulses (occurring in praxis as voltage sparks) can be identified and conditions can be formulated to prevent undesired impulsive effects.

Monitoring the state of the power grid with a limited number of sensors is a fundamental task of grid operators. Reliable information about the grid's state is crucial to ensure reliable and stable power supply as well as the efficient usage of resources. The observer designs from Objective 2 have the potential to lead to novel diagnostic tools for grid operators which directly take into account the algebraic constraints as well as the potential structural changes.

In the presence of an increasing number of volatile power generation (in particular solar and wind power) the stabilization of the power grids gets more and more challenging. The classical stabilization methods of the grid operators are designed under the assumption that there are only a few large power plants which can directly be controlled. These methods will not work anymore in the foreseeable future when the majority of the power generation comes from renewable energy. It is therefore necessary to develop novel methods and control tools for stabilizing the power grid and the results from Objective 3 have a high potential to play a major role in these developments.

In nominal working condition the main focus of grid operators is the efficient usage of the different resources (economic dispatch). Usually these optimizations are based on stationary models, the dynamical effects are not taken into account. The results of Objective 4 have the potential to improve the efficiency of the control methods because they take into account the dynamics of the power grid as well as the effects of (scheduled) structural changes.

The direct applicability of the results of SWITCHEDDAEs are not restricted to power grids. Switched DAEs are a suitable modeling framework for dynamical systems which are subject to sudden structural changes and topology depending algebraic constraints; further examples are analog electrical circuits, water and gas networks, mechanical multibody systems. For all these applications the results of SWITCHEDDAEs will have the potential to lead to novel analysis and control tools.

## Implementation

Objective 5 of SWITCHEDDAEs is specifically dedicated to explore the applicability of the mathematical results obtained in Objectives 1-4.

A special focus will be on deriving a sophisticated power grid model which will go beyond the models currently investigated by the PI [Gross, Trenn, and Wirsén 2014, 2016] in cooperation with the Fraunhofer Institute for Industrial Mathematics (ITWM). In particular, in the current power grid models the only source for dynamics are the generator dynamics; however, electric circuits themselves exhibit dynamic behavior due to inductors and capacitors, which are ignored in the high level models. But each power line has an inductive and capacitive effect, furthermore transformers between the different voltage levels of the power grid are large inductors. As was shown with the academic example in Section 2a, even very trivial circuits with inductors can exhibit possibly disastrous behaviors due to switches. Hence for a complete analysis of power grids with respect to impulsive effects it is necessary to expand the state-of-the-art models to include at least inductive elements. For these more detailed DAE models it will be checked whether the solvability conditions derived in Objective 1 are directly applicable or whether they have to be adjusted.

The observers (Objective 2), stabilizing controllers (Objective 3) and optimal controllers (Objective 4) derived for general DAEs will be adapted to the switched DAE model of power grids. The special structure of the power grid models (e.g. knowledge of the underlying network topology) can be utilized to tailor the general purpose designs of observers and controllers towards the specific needs of power grid operators.

The PI is in contact with the company ABB and will build on this contact to utilize the results of SWITCHEDDAEs in the area of power grids. Furthermore, for the industrial utilization of the results of SWITCHEDDAEs the PI will use the expertise of the Fraunhofer ITWM in the area of power grids and its cooperation with energy companies. The host institution, in particular the Jan C. Willems Center, has currently many projects related to energy networks in cooperations with several companies, e.g. DNV GL and Gastera. The PI will use this opportunity to further utilize the results of SWITCHEDDAEs in industrial applications.

## 2c. Number of words

Section 2a: 3697

Section 2b: 951

## 2d. Literature references

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**Cost estimate**

**3a. Budget**

The overall budget is shown in the table below. Some remarks:

- The PI will spend at least 75% of his time on SWITCHEDDAEs and therefore the whole salary of the PI is charged to the project budget.
- The standard office equipment consists of a stationary computer, however, in order to present results at conferences and workshops, **laptops are necessary** and are therefore charged to the budget.
- The travel cost cover the participation at two international conferences per year for the PI, one participation at an international summer school or conference per year for the PhD-student, one international conference and one national workshop participation per year for the Postdoc. Additional money for inviting at least two international experts per year are included in the budget.

Description		Year 1	Year 2	Year 3	Year 4	Year 5	Total
<b>Staff</b>	<b>FTE Months</b>						
WP	Applicant 1.0 60	76 162	80 124	84 110	88 213	92 483	421 092
WP	PhD Student 1.0 48	20 881	46 105	51 793	55 347	28 892	203 018
WP	Postdoc 1.0 24	0	0	0	64 581	68 327	132 908
<b>Total Staff</b>		<b>97 043</b>	<b>126 229</b>	<b>135 903</b>	<b>208 141</b>	<b>189 702</b>	<b>757 018</b>
Laptop	PI	1 800	0	0	0	0	1 800
Laptop	PhD	1 800	0	0	0	0	1 800
Laptop	Postdoc	0	0	0	1 800	0	1 800
<b>Total Equip.</b>		<b>3 600</b>	<b>0</b>	<b>0</b>	<b>1 800</b>	<b>0</b>	<b>5 400</b>
Travel	PI	3 500	3 500	3 500	3 500	3 500	17 500
Travel	PhD Student	750	1 500	1 500	1 500	750	6 000
Travel	Postdoc	0	0	0	2 000	2 000	4 000
Travel	Guests	2 000	2 000	2 000	2 000	2 000	10 000
<b>Total Travel</b>		<b>6 250</b>	<b>7 000</b>	<b>7 000</b>	<b>9 000</b>	<b>8 250</b>	<b>37 500</b>
<b>Grand total</b>		<b>103 293</b>	<b>133 229</b>	<b>142 903</b>	<b>217 141</b>	<b>197 952</b>	<b>799 918</b>

**3b. Co-financing 'in kind'**

None.

**3c. Co-financing 'in cash'**

None.

**3d. Totals**

Grand total	799 918 EUR
Requested budget	799 918 EUR

**3e. Intended starting date**

01.12.2017

**3f. Have you requested any additional grants for this project either from NWO or from any other institution and/or has the same idea been submitted elsewhere?**

No.

**Curriculum vitae**

**4a. Personal details**

...

**4e. Academic staff supervised**

**PhDs (at TU Kaiserslautern)**

since 2015: Ferdinand Küsters (in cooperation with Fraunhofer ITWM), formal supervisor

since 2014: Rukhsana Kausar, formal supervisor

2014 – 2015: Elisa Mostacciolo (visiting PhD student from University of Benevento, Italy), co-supervisor

2012 – 2016: Andreas Barthlen (in cooperation with Fraunhofer ITWM), formal supervisor

2012 – 2015: Tjorben Groß (in cooperation with Fraunhofer ITWM), formal supervisor

2012: Carmen Pedicini (visiting PhD student from University of Benevento, Italy), co-supervisor

**Postdocs (at TU Kaiserslautern)**

01/09/2014 – 30/11/2015: Aneel Tanwani (since 12/15 a tenured CNRS researcher in Toulouse)

01/03/2016 – 31/08/2016: Deepak Patil (since 11/16 a tenure-track assistant professor at IIT Dehli)

**Total numbers**

PhDs	Total Number	As Promotor	Co-Promotor	Role as co-supervisor
Subtotals PhDs	6	4	0	2
<b>Postdocs</b>				
Subtotals Postdocs	2	formal supervisor at TU Kaiserslautern		
<b>Supervision of Master thesis</b>				
Subtotals Master	4	3 as formal supervisor, 1 as co-supervisor, all at TU Kaiserslautern		
<b>Supervision of Bachelor thesis</b>				
Subtotals Bachelor	9	all as formal supervisors, 7 at TU Kaiserslautern, 2 at TU Ilmenau		

**4f. Brief summary of research over the last five years**

In the last five years my research interest was primarily on the analysis of discontinuous implicit differential equations with a focus on switched DAEs and control theoretical questions. With the introduction of piecewise-smooth distributions as the underlying solution space, I was able to generalize classical results from mathematical control theory to linear switched DAEs including an inverse Lyapunov Theorem [C13]; observability, detectability and controllability characterizations [J17,J14]; duality [J15]; averaging [C16,C24,C27]. I also investigated normal forms for linear nonswitched DAEs like the quasi-Kronecker form based on the Wong sequences approach [J7,J9,J10] and a novel controllability decomposition [J13]. In cooperation with the Fraunhofer ITWM, I have applied the theoretical results to models of electrical power grids and we have obtained solvability and stability results [J16]. Furthermore I am studying the connection between switched DAEs and partial differential equations (PDE): Together with a PhD-student I am investigating how the so-called water hammer effect in water distribution networks (modeled by coupled hyperbolic PDEs) can also be

described by a much simpler switched DAE model. In a recently accepted DFG-proposal I will investigate the coupling of a switched DAE with a hyperbolic PDE with applications in human blood flows.

Another line of research is adaptive nonlinear control with a focus on the so-called funnel controller [J11]. I have proposed a funnel controller based on a very simple switching logic [J12] and the design was also verified in an experimental setup [C10]. Recently, I have used the ideas of funnel control for synchronizing heterogeneous agents [C26].

#### 4g. International activities

I am internationally recognized as the leading expert for switched DAEs. I have given more than 70 scientific talks in 15 different countries and have contacts throughout the world (in particular, USA, Canada, South Korea, Italy, France, England).

##### Longer international research visits

- |                             |  |
|-----------------------------|--|
| <b>2013 (1 month)</b>       | <b>Université de Valenciennes</b> , Valenciennes, France <ul style="list-style-type: none"> <li>- <b>Guest Professor</b> at Université de Valenciennes et du Hainaut-Cambrésis, Valenciennes, France</li> <li>- Lab. d'Automatique, Mécanique et d'Informatique Industrielles et Humaines</li> <li>- Research cooperation with Prof. M. Djemai and Prof. M. Defoort</li> </ul> |
| <b>2009–2010 (9 months)</b> | <b>University of Illinois</b> , Urbana-Champaign, IL, USA <ul style="list-style-type: none"> <li>- Postdoc in "Decision and Control" group of Prof. D. Liberzon</li> <li>- Research in the area of switched dynamical systems</li> </ul>   |
| <b>2004–2005 (6 months)</b> | <b>University of Southampton</b> , Southampton, UK <ul style="list-style-type: none"> <li>- PhD Erasmus student at School of Electronics and Computer Science</li> <li>- Project "Performance and robustness of switched controllers", supervised by Dr. M. French</li> </ul>  |

##### Further international research visits (each about one week):

- 2016**, October (planned), Prof. Hyungbo Shim, **Seoul National University**, Seoul, South Korea
- 2016**, September, Prof. Kanat Camlibel, **University of Groningen**, Groningen, Netherlands
- 2016**, August, Prof. Ricardo Sanfelice, **University of California**, Santa Cruz, CA, USA
- 2016**, March+July, Dr. Aneel Tanwani, **LAAS-CNRS**, Toulouse, Frankreich
- 2015**, November, Prof. Kanat Camlibel, **University of Groningen**, Groningen, Niederlande
- 2015**, March, Prof. Francesco Vasca, **Università degli Studi del Sannio**, Benevento, Italy
- 2014**, October, Prof. Hyungbo Shim, **Seoul National University**, Seoul, South Korea
- 2012**, September, Prof. Mohamed Djemai, **Université de Valenciennes**, Valenciennes, France
- 2012**, August, Dr. Aneel Tanwani, **INRIA Rhône-Alpes**, Grenoble, France
- 2012**, February, Prof. Kanat Camlibel, **University of Groningen**, Groningen, Netherlands
- 2010**, December, Prof. Daniel Liberzon, **University of Illinois**, Urbana-Champaign, IL, USA
- 2010**, September, Prof. Francesco Vasca, **Università degli Studi del Sannio**, Benevento, Italy
- 2010**, July, Prof. Daniel Miller, **University of Waterloo**, Waterloo, ON, Canada
- 2008**, September, Prof. Daniel Liberzon, **University of Illinois**, Urbana-Champaign, IL, USA
- 2006**, March, Nagendra P. Mandalaju, **University of Southampton**, Southampton, UK
- 2003**, August, Prof. Eugene P. Ryan, **University of Bath**, Bath, UK

#### 4h. Other academic activities

##### Editorial Boards

- System & Control Letters, **Associate Editor** (since 2011)
- Springer book series "Differential-Algebraic Equations Forum", **Associate Editor** (since 2012)

##### International Association of Applied Mathematics and Mechanics (Gesellschaft für angewandte Mathematik und Mechanik, GAMM)

- **Vice-Chairman** of GAMM group "Dynamics and Control" (since 2015)
- **Organisation** of GAMM group "Dynamics and Control" meeting, 03/2013, Kaiserslautern
- **Section Chair** (2012), **Minisymposia Organizer** (2012, 2013, 2015) at GAMM Annual Meeting

##### Further organizational duties at conferences and workshops

- 3rd International Conference on Control, Engineering & Information Technology (CEIT'2015), **International Scientific Committee**
- 21st International Symposium on Mathematical Theory of Networks and Systems (MTNS 2014), **Associate Editor**
- Third International Conference on Systems and Control 2013 (ICSC'13), **Associate Editor**
- Conference on Decision and Control (CDC), **Session-Chair** (2010, 2012, 2013, 2015)
- European Control Conference (ECC), **Session-Chair** (2013)

##### Reviewer for journals and conference proceedings

- Applied Mathematics Letters
- Automatica
- Automatisierungstechnik
- Dynamical Systems: An International Journal
- European Journal of Control
- IEEE Transactions on Automatic Control ("Recognized as **Outstanding Reviewer**", 2013)
- IEEE Transactions on Neural Networks
- IET Control Theory & Applications
- IMA Journal of Mathematical Control and Information
- International Journal of Adaptive Control and Signal Processing
- International Journal of Control
- International Journal of Robust and Nonlinear Control
- Journal of Computer and System Sciences
- Journal of Dynamical and Control Systems
- Journal of the Franklin Institute
- Journal of Mathematical Analysis and Applications
- Kybernetika
- Linear Algebra and Its Applications
- Mathematical Problems in Engineering
- Mathematical Methods in the Applied Sciences
- Mathematics of Control, Signals, and Systems
- Mathematics and Computers in Simulation

- Nonlinear Analysis: Hybrid Systems
- SIAM Journal on Control and Optimization
- Systems & Control Letters
- Theoretical Population Biology
- Zentralblatt Mathematik (ZMATH)
- Conferences (ACC, ECC, CCDC, CDC, ICC, ICNAAM, HSCC, MSC, MTNS)

#### 4i. Grants, scholarships and prizes

##### External Funding

- 2016–2019** Coupling hyperbolic PDEs with switched DAEs: Analysis, numerics and application to blood flow models
- PIs: R. Borsche and S. Trenn
  - funded by German Science Foundation (DFG) with ca. 150 000 EUR
  - Part of the DFG priority program SPP 1962 “Non-smooth and complementarity-based distributed parameter systems: Simulation and hierarchical optimization”
- 2014–2016** Observer design for linear switched differential-algebraic equations
- PI: S. Trenn
  - funded by DFG with ca. 162 000 EUR
- 2010–2014** Time-varying and switched differential algebraic equations
- PIs: A. Ilchmann and F. Wirth
  - significant contribution to project proposal and hired as Postdoc within project
  - funded by DFG with ca. 250 000 EUR

##### Awards and Prizes

**STIFT-Preis 2009** für hervorragende anwendungsorientierte Promotions- und Abschlussarbeiten an Thüringer Hochschulen (STIFT-prize for exceptional application orientated PhD and final theses at Thuringian universities), prize money: 1 500 EUR

##### Jugend forscht (Youth researches)

- **Landessieger Thüringen** (First place in the state of Thuringia) **1998** (together with C. Rebel), Topic: “Dreieckskonstruktionen” (Triangular constructions)
- **Landessieger Thüringen** (First place in the state of Thuringia) **1999** (together with C. Rebel), Topic: “Kürschaksche 2n-Ecke” (Kürschaksch’ 2n-gons)

##### Mathematikolympiade (German Mathematics Olympiad)

- **2. Preis Bundesrunde** (Second prize on national level) **1996**
- **1. Preis Landeswettbewerb Thüringen** (First prize on state level) **1996**
- **Anerkennung Landeswettbewerb Thüringen** (Appreciation on state level) **1994, 1998 and 1999**

##### Bundeswettbewerb Mathematik (Federal Mathematics Competition)

- **3. Preis Erste Wettbewerbsrunde** (Third Prize in first round) **1996 and 1997**

## Output

### 5a. Output indicators

#### Author list order

I am following the convention to use *alphabetically ordered* author lists in my publications. In particular, being the first or last author does not have any special significance; in all my publications I made a significant contribution as main researcher or supervisor. Insisting on alphabetical order sometimes led to long discussion with my coauthors from engineering who suggested I should be first author because of my contribution to the corresponding papers.

#### Significance of journal and conference publications

In my research area (mathematical control theory) major research results (usually as a result of one to two years research) are expected to be published in internationally recognized journals (e.g. Automatica; IEEE Transaction on Automatic Control; SIAM Journal on Control and Optimization; International Journal of Control; Systems & Control Letters; International Journal of Robust and Nonlinear Control; Mathematics of Control, Signals, and Systems). However, in contrast to the pure mathematics community, publications in the proceedings of peer reviewed international conferences are also considered important; the most relevant conference in my field is the Conference on Decision and Control (CDC), which usually has an acceptance rate below 50%.

Linear control theory also makes contribution in the field of linear algebra and the most important journals in that area are e.g. Linear Algebra and Its Applications; SIAM Journal on Matrix Analysis and Applications; Linear and Multilinear Algebra.

### 5b. Top publications

My five most significant papers with regard to SWITCHEDDAEs are listed below (numbering according to Section 5c) in chronological order. The reference [J6] is based on my PhD thesis and introduces the space of piecewise-smooth distributions which is fundamental for the solution theory of (linear) switched DAEs and is expected to play an important role also for the nonlinear case. A first extension to the nonlinear (but impulse-free) case was carried out in [J8] and it is expected that many ideas can be borrowed to study the general nonlinear case with Dirac impulses. The decoupling derived in [J9] for linear DAEs into an underdetermined, regular and overdetermined part may be utilized also in the development of a general nonlinear solution theory. For the problem of optimal control it is well-known that the adjoint or dual system plays an important role, hence [J15] is crucial. Finally, the observer design proposed in [J17] for the linear case is a starting point to design a state estimator for general switched DAEs.

[J6] **Regularity of distributional differential algebraic equations** (2009)

S. Trenn, *Mathematics of Control, Signals, and Systems (MCSS)* **21** (3), 229–264

*Abstract:* Time-varying differential algebraic equations (DAEs) of the form  $E\dot{x} = Ax + f$  are considered. The solutions  $x$  and the inhomogeneities  $f$  are assumed to be distributions (generalized functions). As a new approach, distributional entries in the time-varying coefficient matrices  $E$  and  $A$  are allowed as well. Since a multiplication for general distributions is not possible, the smaller space of piecewise-smooth distributions is introduced. This space consists of distributions which could be written as the sum of a piecewise-smooth function and locally finite Dirac impulses and derivatives of Dirac impulses.

A restriction can be defined for the space of piecewise-smooth distributions, this restriction is used to study DAEs with inconsistent initial values; basically, it is assumed that some past trajectory for  $x$  is given and the DAE is activated at some initial time. If this initial trajectory problem has a unique solution for all initial trajectories and all inhomogeneities, then the DAE is called regular. This generalizes the regularity for classical DAEs (i.e. a DAE with constant coefficients). Sufficient and necessary conditions for the regularity of distributional DAEs are given.

- [J8] **Switched nonlinear differential algebraic equations: Solution theory, Lyapunov functions, and stability** (2012)  
 D. Liberzon and S. Trenn, *Automatica* **48** (5), 954–963  
*Abstract:* We study switched nonlinear differential algebraic equations (DAEs) with respect to existence and nature of solutions as well as stability. We utilize piecewise-smooth distributions introduced in earlier work for linear switched DAEs to establish a solution framework for switched nonlinear DAEs. In particular, we allow induced jumps in the solutions. To study stability, we first generalize Lyapunov's direct method to non-switched DAEs and afterwards obtain Lyapunov criteria for asymptotic stability of switched DAEs. Developing appropriate generalizations of the concepts of a common Lyapunov function and multiple Lyapunov functions for DAEs, we derive sufficient conditions for asymptotic stability under arbitrary switching and under sufficiently slow average dwell-time switching, respectively.
- [J9] **The quasi-Kronecker form for matrix pencils** (2012)  
 T. Berger and S. Trenn, *SIAM Journal on Matrix Analysis and Applications* **33** (2), 336–368  
*Abstract:* We study singular matrix pencils and show that the so called Wong sequences yield a quasi-Kronecker form. This form decouples the matrix pencil into an underdetermined part, a regular part and an overdetermined part. This decoupling is sufficient to fully characterize the solution behavior of the differential-algebraic equations associated with the matrix pencil. Furthermore, we show that the minimal indices of the pencil can be determined with only the Wong sequences and that the Kronecker canonical form is a simple corollary of our result, hence, in passing by, we also provide a new proof for the Kronecker canonical form. The results are illustrated with an example given by a simple electrical circuit.
- [J15] **Duality of switched DAEs** (2015)  
 F. Küsters and S. Trenn, *Mathematics of Control, Signals, and Systems* **28** (3), 1–35  
*Abstract:* We present and discuss the definition of the adjoint and dual of a switched differential-algebraic equation (DAE). For a proper duality definition it is necessary to extend the class of switched DAEs to allow for additional impact terms. For this switched DAE with impacts we derive controllability / reachability / determinability / observability characterizations for a given switching signal. Based on this characterizations, we prove duality between controllability / reachability and determinability / observability for switched DAEs.
- [J17] **Determinability and state estimation for switched differential-algebraic equations** (2016)  
 A. Tanwani and S. Trenn, *Automatica*, accepted for publication.  
*Abstract:* The problem of state reconstruction and estimation is considered for a class of switched dynamical systems whose subsystems are modeled using linear differential-algebraic equations (DAEs). Since this system class imposes time-varying dynamic and static (in the form of algebraic constraints) relations on the evolution of state trajectories, an appropriate notion of observability is presented which accommodates these phenomena. Based on this notion, we first derive a formula for the reconstruction of the state of the system where we explicitly obtain an injective mapping from the output to the state. In practice, such a mapping may be difficult to realize numerically and hence a class of estimators is proposed which ensures that the state estimate converges asymptotically to the real state of the system.

## 5c. Output

Refereed journal articles (articles relevant for this proposal are marked with *S*)

- <sup>S</sup>[J17] **Determinability and state estimation for switched differential-algebraic equations** (2016)  
 A. Tanwani and S. Trenn, *Automatica*, accepted for publication.
- <sup>S</sup>[J16] **Solvability and stability of a power system DAE model** (2016)  
 T. Groß, S. Trenn and A. Wirsén, *Systems & Control Letters*, **79**, 12–17
- <sup>S</sup>[J15] **Duality of switched DAEs** (2015)  
 F. Küsters and S. Trenn, *Mathematics of Control, Signals, and Systems* **28** (3), 1–35
- <sup>S</sup>[J14] **Controllability for switched differential-algebraic equations** (2015)  
 F. Küsters, M. G.-M. Ruppert and S. Trenn, *Systems & Control Letters* **78**, 32–39
- <sup>S</sup>[J13] **Kalman controllability decompositions for differential-algebraic systems** (2014)  
 T. Berger and S. Trenn, *Systems & Control Letters* **71**, 54–61
- <sup>S</sup>[J12] **The bang-bang funnel controller for uncertain nonlinear systems with arbitrary relative degree** (2013)  
 D. Liberzon and S. Trenn, *IEEE Transactions on Automatic Control* **58** (12), 3126–3141
- <sup>S</sup>[J11] **Funnel control for systems with relative degree two** (2013)  
 C. Hackl, N. Hopfe, A. Ilchmann, M. Mueller and S. Trenn, *SIAM Journal on Control and Optimization* **51** (2), 965–995
- <sup>S</sup>[J10] **Addition to “The quasi-Kronecker form for matrix pencils”** (2013)  
 T. Berger and S. Trenn, *SIAM Journal on Matrix Analysis and Applications* **34** (1), 94–101
- <sup>S</sup>[J9] **The quasi-Kronecker form for matrix pencils** (2012)  
 T. Berger and S. Trenn, *SIAM Journal on Matrix Analysis and Applications* **33** (2), 336–368
- <sup>S</sup>[J8] **Switched nonlinear differential algebraic equations: Solution theory, Lyapunov functions, and stability** (2012)  
 D. Liberzon and S. Trenn, *Automatica* **48** (5), 954–963
- <sup>S</sup>[J7] **The quasi-Weierstraß form for regular matrix pencils** (2012)  
 T. Berger, A. Ilchmann and S. Trenn, *Linear Algebra and Its Applications* **436** (10), 4052–4069
- <sup>S</sup>[J6] **Regularity of distributional differential algebraic equations** (2009)  
 S. Trenn, *Mathematics of Control, Signals, and Systems (MCSS)* **21** (3), 229–264
- [J5] **A normal form for pure differential algebraic systems** (2008)  
 S. Trenn, *Linear Algebra and Its Applications* **430** (4), 1070–1084
- [J4] **Multilayer Perceptrons: Approximation order and necessary number of hidden units** (2008)  
 S. Trenn, *IEEE Transactions on Neural Networks* **19** (5), 836–844
- [J3] **Pneumatic cylinders: modelling and feedback force-control** (2006)  
 A. Ilchmann, O. Sawodny and S. Trenn, *International Journal on Control* **79** (6), 650–661
- <sup>S</sup>[J2] **Tracking control: performance funnels and prescribed transient behaviour** (2005)  
 A. Ilchmann, E.P. Ryan and S. Trenn, *Systems & Control Letters* **54** (7), 655–670
- [J1] **Input constrained funnel control with applications to chemical reactor models** (2004)  
 A. Ilchmann and S. Trenn, *Systems & Control Letters* **53** (5), 361–375

**Conference proceedings** (fully peer reviewed contributions are marked with \*)

- \*[C33] **Differential-algebraic inclusions with maximal monotone operators** (2016)  
K. Camlibel, L. Iannelli, A. Tanwani and S. Trenn, *Proc. of 55th IEEE Conf. on Decision and Control*, Las Vegas (USA), accepted for publication.
- [C32] **Observer design based on constant-input observability for DAEs** (2016)  
F. Küsters und S. Trenn, to appear in *PAMM 16 (Special Issue: Joint Annual Meeting of GAMM and DMV 2016)*
- [C31] **Stabilization of switched DAEs via fast switching** (2016)  
S. Trenn, to appear in *PAMM 16 (Special Issue: Joint Annual Meeting of GAMM and DMV 2016)*
- \*[C30] **Duality of switched ODEs with jumps** (2015)  
F. Küsters and S. Trenn, *Proc. of 54th IEEE Conf. on Decision and Control*, Osaka (Japan), 4879–4884
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- \*[C5] **On stability of linear switched differential algebraic equations** (2009)  
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- [C3] **Analogue implementation of the funnel controller** (2007)  
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- \*[C2]  **$l^p$  gain bounds for switched adaptive controllers** (2005)  
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 A. Ilchmann, E.P. Ryan and S. Trenn, *Proc. 2004 IEEE Int. Conf. on Control Applications (CCA)*, Taipei, Taiwan, 1032–1036

#### Books and book chapters

- [B6] Chapter 4: **Observability of linear differential-algebraic systems – a survey** (2016)  
 T. Berger, T. Reis and S. Trenn, to appear in *Surveys in Differential-Algebraic Equations IV* (eds. A. Ilchmann and T. Reis), Springer Verlag, ISBN: 978-3-319-46617-0
- [B5] Chapter 8: **Observability of switched linear systems** (2015)  
 M. Petreczky, A. Tanwani and S. Trenn, in: *Hybrid Dynamical Systems* (eds. M. Djemai and M. Defoort), 205–240, Springer Verlag, ISBN: 978-3-319-10794-3
- [B4] Chapter 3: **Stability of switched DAEs** (2013)  
 S. Trenn, in: *Hybrid Systems with Constraints* (eds. J. Daafouz, S. Tarbouriech and M. Sigalotti), 57–81, Wiley, ISBN: 978-1-84821-527-6
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 S. Trenn, in: *Surveys in Differential-Algebraic Equations I* (eds. A. Ilchmann and T. Reis), 137–172, Springer Verlag, ISBN: 978-3-642-34927-0
- [B2] Chapter 6: **Switched differential algebraic equations** (2011) S. Trenn, in *Dynamics and Control of Switched Electronic Systems* (eds. F. Vasca and L. Iannelli), 189–216, Springer Verlag, ISBN: 978-1-4471-2884-7
- [B1] **Distributional differential algebraic equations** (2009)  
 S. Trenn, PhD dissertation, Universitätsverlag Ilmenau, ISBN: 978-3-939473-57-2

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### Selected invited lectures

2016/07/18: **Passive DAEs and maximal monotone operators**

Invited Minisymposium "Control of Differential-Algebraic Equations" (organizer: M. Voigt) at "7th European Congress of Mathematics", Berlin, Germany

2015/11/05: **Funnel synchronization for multi agent systems**

Research seminar (invited by K. Camlibel), University of Groningen, Netherlands

2015/09/18: **Controllability and observability are not dual for switched DAEs**

Invited Minisymposium "Simulation and control of constrained dynamical systems" (organizers: V. Mehrmann and A. Steinbrecher) at International Conference on Scientific Computation And Differential Equations (SciCADE 2015), Potsdam, Germany

2015/03/30+04/01: **Observability/Controllability of switched systems**

Research seminar (invited by F. Vasca), University of Sannio, Benevento, Italy

2014/11/06: **Switched differential algebraic equations: Jumps and impulses**

Lothar-Collatz-Kolloquium für Angewandte Mathematik (invited by T. Reis), Hamburg, Germany

2014/10/23: **Basics on differential-algebraic equations**

Invited tutorial at "14th International Conference on Control, Automation and Systems (ICCAS 2014)", Goyang, South Korea

2014/10/15-17: **Short Course on Differential-Algebraic Equations**

Seminar at Seoul National University (invited by H. Shim), Seoul, South Korea

2014/06/05: **Stability of switched DAEs**

Research Seminar Control events at Lyon (invited by V. Andrieu), University of Lyon, Lyon, France

2014/03/11: **Controllability notions for switched DAEs**

Invited Minisymposium "Control of Differential-Algebraic Equations" (organizers: U. Konigorski and T. Reis) at GAMM Annual Meeting 2014, Erlangen, Germany

2012/11/28: **Der Funnel-Regler: 10 Jahre Adaption**

Research seminar (invited by J. Lunze), Ruhr-Universität Bochum, Germany

2012/10/01: **The bang-bang funnel controller**

Invited Lecture at Workshop "Event-Based Control and Optimization", München, Germany

2012/09/20: **Switched differential algebraic equations: Jumps and impulses**

Research seminar (invited by M. Djemai), University of Valenciennes, France

2012/06/18: **The joint spectral radius for semigroups generated by switched differential algebraic equations**

Invited Minisymposium "Numerical algorithms for switching systems: from theory to applications" (organizers: N. Guglielmi and R. Jungers) at "SIAM Conference on Applied Linear Algebra 2012", Valencia, Spain

2012/06/04: **Stability of switched DAEs**

Invited lecture at "Workshop Architecture Hybride et Contraintes (ArHyCo)", Paris, France

**2012/02/21: Switched differential algebraic equations: Jumps and impulses**

Research seminar (invited by K. Camlibel), University of Groningen, Groningen, The Netherlands

**2010/09/17: Modeling electrical circuits with switched differential algebraic equations**

Research seminar (invited by F. Vasca), Universit degli Studi del Sannio, Benevento, Italy

**2010/07/29: Switched differential algebraic equations: Solution theory, Lyapunov functions, and stability**

Invited lecture at "2010 Workshop on Hybrid Dynamic Systems", Waterloo, Canada

**2010/07/27: The bang-bang funnel controller**

Research seminar (invited by D. Miller), University of Waterloo, Waterloo, Canada

**2010/01/07: Differential algebraic equations and distributional solutions**

Research seminar (invited by S. Siegmund), Technische Universität Dresden, Dresden, Germany

**2009/07/14: Differential-algebraische Gleichungen: Ein distributioneller Ansatz**

Research seminar (invited by F. Colonius), Universität Augsburg, Augsburg, Germany

**2009/06/23: Eine Lösungstheorie für geschaltete differential-algebraische Gleichungen**

Research seminar (invited by W. Seiler), Universität Kassel, Kassel, Germany

**2008/09/10: Solution theory for switched differential-algebraic equations**

Research seminar (invited by D. Liberzon), University of Illinois at Urbana-Champaign, Urbana-Champaign, USA

**2007/06/20: Linear differential-algebraic equations with piecewise smooth coefficients**

Invited Session "Random and non-autonomous dynamical systems" (organizers: R. Johnson and H. Crauel) at "Joint International Meeting UMI - DMV", Perugia, Italy

**2006/11/02: Distributional DAEs**

Research seminar (invited by V. Mehrmann), Technische Universität Berlin, Berlin, Germany

For a complete list of my over seventy **scientific talks** including slides see my webpage  
<http://research.stephantrenn.de/index.php/talks.html>