# Switch induced instabilities for stable power system DAE models

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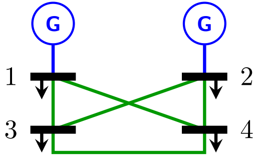
# Power systems model

Power grid consists of

- $n_g \in \mathbb{N}$ generators
- power lines
- $n_g + n_b$ line connectors (**busses**)
- power demand at each bus




## Variables

For each generator:

- $\alpha(t)$ and $\omega(t)$ angle and angular velocity of rotating mass
- $P_g(t)$ generator power acting on turbine

For each bus:

- $V(t)$ and $\theta(t)$ voltage modulus and angle
- $P(t)$, $Q(t)$ active and reactive power demand

# Basic modelling assumptions

## Generator

- Rotating mass(es) with linear friction (and linear elastic coupling)
- Constant voltage behind transient reactance model (*Kundur 1994*)
- $\sin(\alpha(t) - \theta(t)) \approx \alpha(t) - \theta(t)$

## Busses

- $V(t) \approx 1$ (per unit)
- $\sin(\theta_i - \theta_j) \approx \theta_i - \theta_j$ for any adjacent busses $i$ and $j$

## Lines

$\Pi$-model with negligible conductances

$\hookrightarrow$ reactive power flow can be ignored

# Linearized model

## Dynamics of $i$-th generator

$$\dot{\alpha}_i(t) = \omega_i(t)$$
$$m_i\dot{\omega}_i(t) = -D_i\omega(t) - P_{e,i}(t) + P_{g,i}(t)$$

where $P_{e,i}(t) = \frac{1}{z_i}(\alpha_i(t) - \theta_i(t))$ and $m_i > 0$ is the moment of inertia

## Linearized power flow balance at each bus $i$

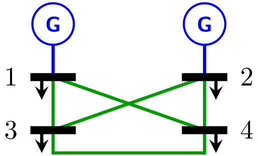
$$0 = P_i(t) + P_{e,i}(t) - \sum_{j=1}^{n_g+n_b} \ell_{ij}(\theta_i(t) - \theta_j(t)),$$

where $\ell_{ij} = \ell_{ji} \geq 0$ is the line susceptance and $P_{e,i}(t) = 0$ for $i > n_g$

# Linear DAE model

Overall we get a linear DAE

$$E\dot{x} = Ax + Bu$$




where in our example

$$x = (\alpha_1, \alpha_2, \omega_1, \omega_2, \theta_1, \theta_2, \theta_3, \theta_4)^\top$$

$$u = (P_{g,1}, P_{g,2}, P_1, P_2, P_3, P_4)^\top$$

and, with $\ell_{ii} := \sum_{j=1}^{4} \ell_{ij}$,

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -z_1^{-1} & 0 & -D_1 & 0 & z_1^{-1} & 0 & 0 & 0 \\ 0 & -z_2^{-1} & 0 & -D_2 & 0 & z_2^{-1} & 0 & 0 \\ z_1^{-1} & 0 & 0 & 0 & -z_1^{-1}-\ell_{11} & 0 & \ell_{13} & \ell_{14} \\ 0 & z_2^{-1} & 0 & 0 & 0 & -z_2^{-1}-\ell_{22} & \ell_{23} & \ell_{24} \\ 0 & 0 & 0 & 0 & \ell_{31} & \ell_{32} & -\ell_{33} & \ell_{34} \\ 0 & 0 & 0 & 0 & \ell_{41} & \ell_{42} & \ell_{43} & -\ell_{44} \end{bmatrix}$$

# General DAE-structure

DAE-model for $n_g$ generators and $n_b$ busses has the following structure:

$$E\dot{x} = Ax + Bu \qquad \textbf{(powerDAE)}$$

with

$$x = (\alpha_1, \ldots, \alpha_{n_g}, \omega_1, \ldots, \omega_{n_g}, \theta_1, \theta_2, \ldots, \theta_{n_g+n_b})^\top$$
$$u = (P_{g,1}, \ldots, P_{g,n_g}, P_1, \ldots, P_{n_g+n_b})^\top$$

and

$$E = \begin{bmatrix} I_{n_g} & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & I_{n_g} & 0 \\ -Z^{-1} & -D & \begin{bmatrix} Z^{-1} & 0 \end{bmatrix} \\ \begin{bmatrix} Z^{-1} \\ 0 \end{bmatrix} & 0 & -\mathfrak{L}-\begin{bmatrix} Z^{-1} & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ I_{n_g} & 0 \\ 0 & I_{n_g+n_b} \end{bmatrix}$$

where $\mathfrak{L} = [\ell_{ij}]$ is the (weighted) **Laplacian matrix** of the network

# Solvability and Stability

## Theorem (Solvability and Stability, *Groß et al. 2016*)

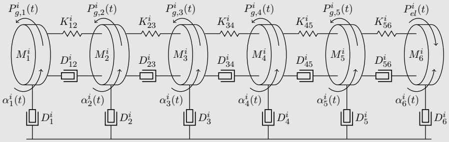
*Consider a power grid network and assume that it is connected. Then*

- *(**powerDAE**) is regular, i.e. existence and uniqueness of solutions guaranteed*
- *(**powerDAE**) has index one, i.e. it is numerically well posed*
- *(**powerDAE**) is stable, i.e. all solutions remain bounded*

T.B. Gross, S. Trenn, A. Wirsen: Solvability and stability of a power system DAE model, *Syst. Control Lett.*, 29 , pp. 12–17, 2016.

## Remark

Result remains true for multiple-rotating mass models of generators.

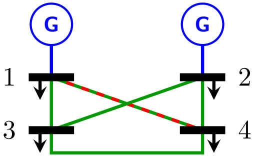


Modelling

**Instability due to switching**

Sufficient condition for stability under arbitrary switching

# Topological changes




$$E_1\dot{x} = A_1 x + B_1 u \quad \text{in mode 1}$$
$$E_2\dot{x} = A_2 x + B_2 u \quad \text{in mode 2}$$

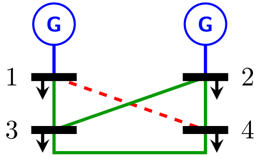
or, introducing a switching signal $\sigma : \mathbb{R} \to \{1,2\}$

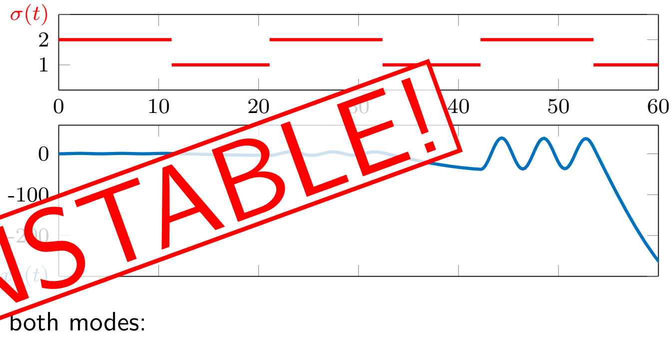
$$E_{\sigma(t)}\dot{x} = A_{\sigma(t)} x + B_{\sigma(t)} u$$

In fact, topological changes (removal / addition / parameter changes of lines) only effect Laplacian matrix $\mathfrak{L}$!

$$E = \begin{bmatrix} I_{n_g} & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_{\sigma(t)} = \begin{bmatrix} 0 & I_{n_g} & 0 \\ -Z^{-1} & -D & \begin{bmatrix} Z^{-1} & 0 \end{bmatrix} \\ \begin{bmatrix} Z^{-1} \\ 0 \end{bmatrix} & 0 & -\mathfrak{L}_{\sigma(t)} - \begin{bmatrix} Z^{-1} & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ I_{n_g} & 0 \\ 0 & I_{n_g+n_b} \end{bmatrix}$$

# Simulation




Parameters:

$m_1 = m_2 = 1$

$z_1 = z_2 = 0.1$

$D_1 = D_2 = 0.01$

and Laplacian-matrices for both modes:

$$\mathfrak{L}_1 = \begin{bmatrix} -0.01 & 0 & 0.005 & 0.005 \\ 0 & -5.005 & 0.005 & 5 \\ 0.005 & 0.005 & -0.02 & 0.01 \\ 0.005 & 5 & 0.01 & -5.015 \end{bmatrix}, \quad \mathfrak{L}_2 = \begin{bmatrix} -2.005 & 0 & 0.005 & 2 \\ 0 & -5.005 & 0.005 & 5 \\ 0.005 & 0.005 & -0.02 & 0.01 \\ 2 & 5 & 0.01 & -7.01 \end{bmatrix}$$

Modelling

Instability due to switching

**Sufficient condition for stability under arbitrary switching**

# Stability and Lyapunov functions

$$E_\sigma \dot{x} = A_\sigma x \qquad \textbf{(swDAE)}$$

## Theorem (cf. *Liberzon and T. 2012*)

*Assume (**swDAE**) to be regular and index one with consistency projectors $\Pi_p$.If*

1. *each mode is stable with Lyapunov function $V_p(\cdot)$*
2. $V_q(\Pi_q x) \leq V_p(x)$ *for all $p, q$ and all $x \in \operatorname{im} \Pi_p$*

*then (**swDAE**) is stable under arbitrary switching.*

D. Liberzon, S. Trenn: Switched nonlinear differential algebraic equations: Solution theory, Lyapunov functions, and stability. *Automatica*, 48 (5), pp. 954–963, 2012.

## Remark

If $E$-matrix is switch-independent and has the form $E = \begin{bmatrix} E_1 & 0 \\ 0 & 0 \end{bmatrix}$ with invertible $E_1$, then $V_q(\Pi_q x) = V_q(x)$ for all $x \in \operatorname{im} \Pi_p$.

$\hookrightarrow$ common Lyapunov function guarantees stability

# Key lemma

## Lemma

*Consider (E,A) with structure*

$$E = \begin{bmatrix} E_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} A_1 & A_2 & 0 \\ A_3 & -\mathfrak{L}_1 + A_4 & -\mathfrak{L}_2 \\ 0 & -\mathfrak{L}_3 & -\mathfrak{L}_4 \end{bmatrix},$$

*where* $\mathfrak{L} = \begin{bmatrix} \mathfrak{L}_1 & \mathfrak{L}_2 \\ \mathfrak{L}_3 & \mathfrak{L}_4 \end{bmatrix}$ *is a (weighted) Laplacian matrix. If*

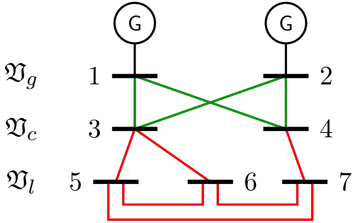
- $(E, A)$ *is regular, index one and stable*
- $\operatorname{rank} \mathfrak{L}_3 = 1$

*then* $\exists$ *common Lyapunov function for all possible* $\mathfrak{L}_4$

# Structural assumption for stability

Assume $\mathfrak{V} = \mathfrak{V}_g \dot{\cup} \mathfrak{V}_c \dot{\cup} \mathfrak{V}_l$ such that

1. $\mathfrak{V}_g$ are the generator busses
2. no edges between $\mathfrak{V}_g$ and $\mathfrak{V}_l$
3. full connection between $\mathfrak{V}_g$ and $\mathfrak{V}_c$
4. Laplacian of edges between $\mathfrak{V}_g$ and $\mathfrak{V}_c$ has rank one
5. topological changes only occur in edges in $\mathfrak{V}_c \cup \mathfrak{V}_l$




## Theorem

*Under above assumptions, stability is preserved under arbitrary switching.*

T.B. Gross, S. Trenn, A. Wirsen: Switch induced instabilities for stable power system DAE models. *Proceedings of IFAC Conference on Analysis and Design of Hybrid Systems (ADHS 2018)*, Oxford, UK, IFAC-PapersOnLine 51(16), pp. 127–132, 2018.

# Summary

- Presentation of a simple linear DAE-model for power grids
- This DAE model is regular, index 1 and stable
- Sudden repeated changes in line parameter may lead to instability
- Topological conditions are presented which prevent instability

## Open questions

- Physical interpretation
- Nonlinear and more detailed model