DFG Proposal
Observer design for linear switched differential-algebraic equations

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Summary

A dynamical system comprises a mathematical model of an underlying physical phenomenon. It has two basic components: *external signals* which interconnect the system with its environment and the internal *state* that evolves according to the model description. The external signals can usually further be split into inputs and outputs. One of the basic problem associated with any dynamical system is that of constructing an *observer* which uses the available information of the external signals to estimate the internal state. The purpose of this project is to develop observers for dynamical systems modeled as *switched differential-algebraic equations* (DAEs). The motivation to study this particular system class is twofold: 1) In contrast to ordinary differential equations (ODEs), DAEs include differential as well as algebraic equations. Practically every system's model contains algebraic equations in the first place so it is natural to use DAEs (instead of the simplified ODEs) as a starting point. 2) Possible structural changes (like switches in electrical circuits or component faults in general physical system) can be modeled within the framework of switched systems. As an application of the proposed project, consider for example (national) electrical grids, which are large electrical circuits modeled as DAEs. An observer would then be used to monitor the energy flows through the transmission lines and could prevent overloading. Sudden structural changes in electrical grids are common and have to be taken into account; examples are: tripping of power lines due to harsh weather conditions, or a sudden drop in the energy production by wind turbines when whole wind parks are switched off in the presence of too strong winds. Hence a possible application of the theoretical results obtained by the proposed project could be improved monitoring tools for electrical grids.

Project description

1 State of the art and preliminary work

First principle modeling of physical phenomena often leads to a mathematical description consisting of differential equations as well as algebraic constraints or, in other words, *differential-algebraic equations* (DAEs). If the physical systems also exhibits sudden structural changes (for example switches in electrical circuits or component faults in general) a common mathematical description is via *switched systems*. With this motivation, we propose the study of *switched linear DAEs* of the following form:

$$\begin{aligned} E_\sigma \dot{x} &= A_\sigma x + B_\sigma u \\ y &= C_\sigma x + D_\sigma u. \end{aligned} \tag{1}$$

This proposal aims at studying the structural properties that relate to **extracting information** about the state x and the mode σ from the knowledge of the output y. The first one, commonly called (state) *observability*, is based on investigating the mapping that exists between the state space and the output space. The second property, called *mode observability*, relates to the mapping between the switching signals and the output space. On an abstract level, the fundamental question in studying either of these properties is, whether the underlying mapping is injective; this characteristic determines whether the output can reveal complete information about the state or the current mode. Both these system properties reveal fundamental characteristics of switched systems, in the spirit of what one can say about the qualitative behavior of the system in the long run or how much one can infer from and influence the systems behavior based on observed data. Also, because of the switching dynamics, it is observed that the interchange between discrete and continuous dynamics not only reveals several novel phenomena, but also provides some new insights into the structure of switched systems.

However, the most significant aspect of the information extracting properties, in addition to being theoretically rich, is their utility in solving some of the prominent design problems. Our earlier work on switched systems – relating to the properties of stability [25], and observability [39] – addresses the problems from analytical perspective, and in this proposal we make the transition from the analysis to design problems. The observability property allows for state estimation from the measured outputs by designing appropriately an auxiliary dynamical system, called *observer*; and the concepts related to mode observability of switched systems are utilized for example in fault detection.

Studying the design problems in the context of switched DAEs has not received much attention in the literature and this proposal develops the framework for exploring a wide range of system design problems with switching dynamics. The next section gives an overview of the results which have been obtained so far by other researchers; afterwards, the corresponding preliminary work of the principal investigator are summarized.

Results obtained by other workers in that area

Observability and observer design are classical problems in systems theory and the earliest solution of these problems for linear time invariant systems date back to the early 1960's by Kalman [19] and Luenberger [27, 28]. Since then, the problem has been addressed for various other types of dynamical systems. Several classes of nonlinear dynamical systems provided the first extension, e.g., [8, 15, 14, 17, 21, 22, 48], and several generalizations of such systems are still being studied.

In the context of (nonswitched) DAEs, observer design methods were initially studied e.g. in [10, 13]. In contrast to ODEs, the observer design in DAEs requires additional structural assumptions and, furthermore, the order of the observer may depend on the design method. Because of these added generalities, observer designs for nonswitched DAEs are still being studied [9, 11].

During the past decade, however, the focus has shifted towards the study of observability and observer design for nonsmooth dynamical systems since they generalize a large number of physical and digitally-interfaced models, e.g. [3, 32, 46]. Out of several existing formalisms for modeling nonsmooth behavior, switched systems form an important subclass which comprise a family of subsystems and a switching rule that determines the active subsystem [23]. The presence of a switching signal brings an extra dimension to the problem of observability for such systems. Observability and observer design for switched linear ODEs with unknown switching signal (or discrete state) were studied by [46, 2]. Assuming that the individual subsystems are observable, algorithms are proposed for computing the continuous as well as the discrete state. However, if the switching signal is known, then without requiring the observability of individual subsystems, the conditions based on gathering information about the continuous state from individual subsystems (without addressing observer construction) appear in [31, 47].

From an application's viewpoint, the state estimated by the observer may be used in the construction of state feedback laws. In linear time-invariant systems, due to the separation principle, this is achieved easily [28]. In nonlinear systems, several results on this problem have appeared in [1, 20, 41]. Observers have also been immensely utilized in model-based fault detection, e.g., [4, 18, 29, 16]. However, in switched systems (even for case of ODE-subsystems) such applications have not yet been studied.

Own results in connection with the proposed project

For carrying out the research of the proposed project it is planned to fill the requested postdoc position by Aneel Tanwani (AT); therefore, his contributions to the field are given alongside the contributions by the principal investigator (ST).

Switched DAEs

The mathematical framework of (linear) switched DAEs was introduced by ST [42, 44]. The mathematical difficulties of treating these kind of systems lies in the fact that no classical solutions exists; jumps and even Dirac impulses may occur in the solution. This made it necessary to introduce a suitable distributional solution space to handle switched DAEs in a mathematically rigorous way. The further development of this distributional solution theory was one of the main purposes of the recent DFG-project Wi1458/10-1 entitled "Zeitvariante und geschaltete differential-algebraische Gleichungen" (Time-varying and switched DAEs) in which ST held a postdoc position until November 2011. Questions of observability and observer design were not part of that project.

Using an observer for feedback stabilization raises the question of stability of the closed loop. Hence the recent stability results for switched DAEs [24, 26, 25, 45] obtained by ST and coworkers are important preliminary results for the project.

Observer design for switched systems

Based on the viewpoint from the above mentioned works [31, 47], a unified approach towards observability and observers in a more general framework is studied in the recent papers [36, 37] by AT and coworkers. In contrast to the classical approach, in [37] impulsive observers have been employed in order to compensate for the lack of complete information about the state at each time instant. Following this line of thought, AT and coworkers have extended the methods towards nonlinear switched systems [30] by proposing sufficient geometric conditions for observability that lead to a design of impulsive/hybrid-type observers.

In line with the project proposal, AT and ST recently started studying the observability of switched DAEs [38, 39] and were able to give a complete characterization of observability for (linear) switched DAEs in case the switching signal is known. They also obtained sufficient and necessary conditions for observability in the case that the switching signal is only partially known. An observer design for switched DAEs was suggested by ST and AT in the very recent conference contribution [40], the formal proofs and further details are not worked out yet and are an important part of the project proposal.

Normal forms for DAEs

Normal and reduced forms play an important role for the design and analyis of observers, hence it is to be expected that the normal form for so called pure DAEs (nonswitched) obtained by ST [43] is an important preliminary work. Also the quasi-Weierstrass and quasi-Kronecker form for (nonswitched) DAEs studied by ST and coworkers [5, 6, 7] will be relevant, the former already played a prominent role in the observability characterization [38, 39] mentioned above.

Invertibility, mode and fault detection

A question related to observability is, whether it is possible to deduce the input from the observed output. Viewing the switching signal as an external input this also covers the important problem of mode detection. These kind of questions for switched ODEs were studied by AT in his PhD-thesis [32], see also [34, 35]. An application of this theory can be seen in the fault detection in switching electrical networks [33], however generalizing this approach to switched DAEs remains to be done. First results for switched DAEs in this context were obtained by ST [12] where a method was presented to check whether switches or component faults can produce "spikes" in the internal states possibly leading to more damage.

2 Objectives and work programm

2.1 Objectives

Our aim is the development of methods for extracting internal information of a system where only a limited number of measurements are available. In particular we want to construct a **state observer** for the class of switched DAEs. A strong motivation for this construction is the necessity to monitor the internal states of power networks with only a (relatively) few sensors and our hope is that the results obtained in the proposed project can contribute to the solution of the challenges of the future energy generation and distribution. The first main goal of the proposal is therefore the extension of our preliminary observer design reported in [40] under more realistic assumptions like uncertainties in the measurements or for the case that the overall system is only detectable (and not observable). The second main goal of our proposal is the development of methods for **mode detection**, which can be interpreted as a "mode observer". The application of mode detection is twofold: 1) The above state observer relies on the knowledge of the switching signal, hence an additional mode observer is necessary if the switching signal is not known; 2) by including faults as additional modes in the switched system model, mode detection can directly be used for fault detection.

Finally, we would like to stress that we only consider *linear* switched DAEs. We are aware that more sophisticated models of physical systems are usually nonlinear; however, for nonlinear switched DAEs the solution theory is much more involved and is not fully understood yet. Hence in order to study observer designs for nonlinear switched DAEs one first has to establish a suitable solution theory for nonlinear switched DAEs and this is outside the scope of this project and may be the objective of future project proposal. Nevertheless, it should be mentioned that the proposed observer is expected to be highly nonlinear so that the resulting closed loop is already a nonlinear switched DAE.

2.2 Work program incl. proposed research methods

I. Observer design for linear switched DAE (6 months)

Based on the observability notions for switched DAEs (1) developed in [39] observers will be designed under the assumption that the switching signal is known; in other words, we view (1) as a time-varying linear system. The major difficulty lies in the fact, that (1) can be observable over a certain time interval although none of the individual modes need to be observable in the classical sense. As a result, in each mode only parts of the state can be (approximately) determined and the overall state estimation error cannot be expected to decrease monotonically. Further difficulties lie in the fact [38] that for each switching instance four independent observability subspaces have to be taken into account. These four subspaces play a crucial role in the observer design, as each space leads to partial state estimations which first have to be combined locally for each mode and afterwards the partial state estimations have to be combined for several consecutive modes.

Using this intuition, some preliminary observer design is given in our recent conference paper [40], where our approach is to design a so-called impulsive observer in the sense that the proposed observer consists of a system copy of the form

$$E_\sigma \dot{\hat{x}} = A_\sigma \hat{x} + B_\sigma u$$

together with a correction of $\hat{x}$ at each switching instance. The calculation of the correction term is therefore the key ingredient of our observer; it takes into account the algebraic constraints imposed by the DAE in each mode, an estimate of the observable part of the state via a classical Luenberger observer and, finally, the presence of Dirac impulses in the observed output. The latter is an outstanding property of our proposed observer as the presence of Dirac impulses in the output is a feature only present in switched DAEs and the utilization of this knowledge is necessary for the observer design. It should be noted that the presence of Dirac impulses is due to an higher index of the DAEs describing the individual modes. In particular, we do not assume that each mode is index one; in fact, since the observer utilizes the information from the Dirac impulses a higher index is actually beneficial (at least from a theoretical point of view).

In this part of the work program we would like to provide further details of the preliminary observer design proposed in [40] and also study questions like robustness with respect to measurement noise as well as uncertainties in the switching signal. Furthermore, we have to address the problem of measuring Dirac-impulses which is necessary in theory; but in practice there do not exist pure Dirac impulses, only "smeared out" versions of it. While the presence of higher derivatives of Dirac impulses (induced by higher indeces of the underlying DAEs) allows for more information extraction in theory, it is more difficult to measure these higher derivatives of Dirac impulses in reality. This has to be taken into account in the corresponding analysis.

As mentioned above, in order to estimate a certain part of the state we are using a classical Luenberger observer. We would also like to investigate how the overall observer design benefits from the usage of a more sophisticated (possibly nonlinear) observer instead of the Luenberger observer.

II. Detetectability and observers (9 months)

It is well known from classical systems theory that for the existence of an observer detectability of the system is sufficient and necessary. A characterization of detectability of switched DAEs is not available yet and sufficient and necessary conditions have to be obtained first. Furthermore, it would be of interest to formulate an equivalence between (a certain kind of) detectability and the existence of an observer (not necessarily in a constructive way).

Afterwards, it has to be investigated how the observer design from above has to be modified if the switched DAE is only detectable. Since the above observer design relies on the fact that the observable part can be estimated arbitrarily well in arbitrarily short time intervals, it can not be expected that the above observer works without further assumptions in the detectable case. One possible way to overcome this problem is to make an assumption on the minimum dwell time of the switching signal (which was not necessary above). However, if the switching times are too far apart the state estimation error in the undetectable part of each mode might get unacceptably large. Therefore, we have two competing goals: 1) For the estimation of the detectable part we have to remain in each mode for a certain amount of time (depending on the unobservable eigenvalues), resulting possibly in large dwell times; 2) We have to switch sufficiently fast, so that the undetectable part of the estimation can be corrected with the information coming from the switches (small dwell times). The study of this interplay between too large and too small dwell times is a major challenge in this part of the work program.

III. Mode detection for linear switched DAEs (9 months)

In the previous two parts of the work program, it was assumed that the switching signal is known; however, if the switches are induced e.g. by component faults this assumption is not valid anymore. Therefore a relevant question strongly related to observability is the possibility to detect the current mode of the switched system. Furthermore, with a reliable mode detection, switched DAEs (1) with **state-dependent switching** can be handled, because the mode detection can extract the time-varying switching signal which can then be used in the above observer design.

The problem of mode detection has only been studied recently for switched ODEs. The common approach is to assume that the initial state and the outputs are known, and for each subsystem a mapping is constructed that relates the outputs and the initial state of the system. Under the assumption that the range space of such mappings do not coincide for any of the constituent subsystems, one indeed recovers the mode of the system. This approach has two major drawbacks: 1) The initial state has to be known, 2) The derivatives of the output have to be evaluated. Nevertheless, it is of theoretical interest whether this approach can be extended to switched DAEs. In fact, due to the different algebraic constraints in each mode of the switched DAE it might in fact be easier to detect the mode.

Ultimately, our aim is to extract the mode without the knowledge of the initial value of the state variable (otherwise we cannot use the mode detection in combination with an observer). In general, this will not be possible, but we would like to find checkable condition when this is possible in theory and also how the corresponding mode detection can be implemented.

Another relevant question is the presence of uncertainties in the output measurements and how they influence the capabilities in detecting the active mode.

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