Instability in Power Systems due to Switching

Stephan Trenn

Jan C. Willems Center for Systems and Control
University of Groningen, Netherlands

Joint work with **Tjorben Groß** and **Andreas Wirsen**
Fraunhofer ITWM, Kaiserslautern, Germany

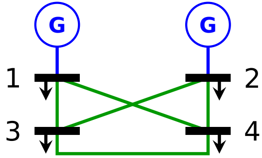
This work was partially supported by the Fraunhofer Internal Programs under Grant No. Discover 828378 and by NWO Vidi grant 639.032.733

GAMM Annual Meeting 2018, Munich, Germany, 21.03.2018, 17:30

Power systems model

Power grid consists of

- $n_g \in \mathbb{N}$ generators
- power lines
- $n_g + n_b$ line connectors (**busses**)
- power demand at each bus



Variables

For each generator:

- $\alpha(t)$ and $\omega(t)$ angle and angular velocity of rotating mass
- $P_g(t)$ generator power acting on turbine

For each bus:

- $V(t)$ and $\theta(t)$ voltage modulus and angle
- $P(t)$, $Q(t)$ active and reactive power demand

Basic modelling assumptions

Generator

- Rotating mass(es) with linear friction (and linear elastic coupling)
- Constant voltage behind transient reactance model (*Kundur 1994*)
- $\sin(\alpha(t) - \theta(t)) \approx \alpha(t) - \theta(t)$

Busses

- $V(t) \approx 1$ (per unit)
- $\sin(\theta_i - \theta_j) \approx \theta_i - \theta_j$ for any adjacent busses i and j

Lines

Π-model with negligible conductances

$\hookrightarrow$ reactive power flow can be ignored

Linearized model

Dynamics of i-th generator

$$\dot{\alpha}_i(t) = \omega_i(t)$$
$$m_i \dot{\omega}_i(t) = -D_i \omega(t) - P_{e,i}(t) + P_{g,i}(t)$$

where $P_{e,i}(t) = \frac{1}{z_i}(\alpha_i(t) - \theta_i(t))$ and $m_i > 0$ is the moment of inertia

Linearized power flow balance at each bus i

$$0 = P_i(t) + P_{e,i}(t) - \sum_{j=1}^{n_g+n_b} \ell_{ij}(\theta_i(t) - \theta_j(t)),$$

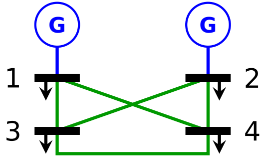
where $\ell_{ij} = \ell_{ji} \geq 0$ is the line susceptance and $P_{e,i}(t) = 0$ for $i > n_g$

Linear DAE model

Overall we get a linear DAE

$$E\dot{x} = Ax + Bu$$

where in our example



$$x = (\alpha_1, \alpha_2, \omega_1, \omega_2, \theta_1, \theta_2, \theta_3, \theta_4)^\top$$
$$u = (P_{g,1}, P_{g,2}, P_1, P_2, P_3, P_4)^\top$$

and, with $\ell_{ii} := \sum_{j=1}^{4} \ell_{ij}$,

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -z_1^{-1} & 0 & -D_1 & 0 & z_1^{-1} & 0 & 0 & 0 \\ 0 & -z_2^{-1} & 0 & -D_2 & 0 & z_2^{-1} & 0 & 0 \\ z_1^{-1} & 0 & 0 & 0 & -z_1^{-1}-\ell_{11} & 0 & \ell_{13} & \ell_{14} \\ 0 & z_2^{-1} & 0 & 0 & 0 & -z_2^{-1}-\ell_{22} & \ell_{23} & \ell_{24} \\ 0 & 0 & 0 & 0 & \ell_{31} & \ell_{32} & -\ell_{33} & \ell_{34} \\ 0 & 0 & 0 & 0 & \ell_{41} & \ell_{42} & \ell_{43} & -\ell_{44} \end{bmatrix}$$

General DAE-structure

DAE-model for n_g generators and n_b busses has the following structure:

$$E\dot{x} = Ax + Bu \qquad (\textbf{powerDAE})$$

with

$$x = (\alpha_1, \ldots, \alpha_{n_g}, \omega_1, \ldots, \omega_{n_g}, \theta_1, \theta_2, \ldots, \theta_{n_g+n_b})^\top$$
$$u = (P_{g,1}, \ldots, P_{g,n_g}, P_1, \ldots, P_{n_g+n_b})^\top$$

and

$$E = \begin{bmatrix} I_{n_g} & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & I_{n_g} & 0 \\ -Z^{-1} & -D & \begin{bmatrix} Z^{-1} & 0 \end{bmatrix} \\ \begin{bmatrix} Z^{-1} \\ 0 \end{bmatrix} & 0 & -\mathfrak{L}-\begin{bmatrix} Z^{-1} & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ I_{n_g} & 0 \\ 0 & I_{n_g+n_b} \end{bmatrix}$$

where $\mathfrak{L} = [\ell_{ij}]$ is the (weighted) **Laplacian matrix** of the network

Solvability and Stability

Theorem (Solvability and Stability, *Groß et al. 2016*)

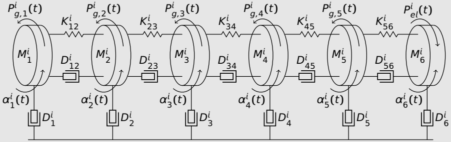
Consider a power grid network and assume that is connected. Then

- *(**powerDAE**) is regular, i.e. existence and uniqueness of solutions*
- *(**powerDAE**) has index one, i.e. it is numerically well posed*
- *(**powerDAE**) is stable, i.e. all solutions remain bounded*

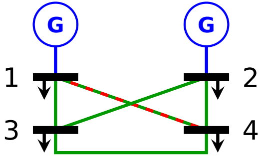
T.B. Gross, S. Trenn, A. Wirsen: Solvability and stability of a power system DAE model, *Syst. Control Lett.*, 29 , pp. 12–17, 2016.

Remark

Result remains true for multiple-rotating mass models of generators.



Topological changes



$$E_1\dot{x} = A_1 x + B_1 u \quad \text{in mode 1}$$
$$E_2\dot{x} = A_2 x + B_2 u \quad \text{in mode 2}$$

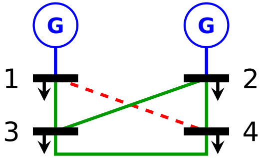
or, introducing a switching signal $\sigma : \mathbb{R} \to \{1,2\}$

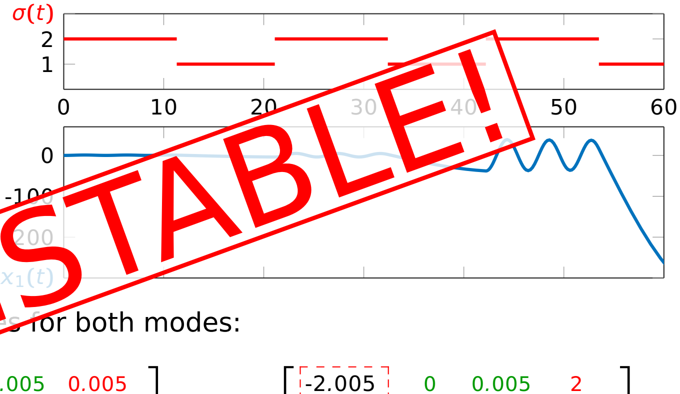
$$E_{\sigma(t)}\dot{x} = A_{\sigma(t)} x + B_{\sigma(t)} u$$

In fact, topological changes (removal / addition / parameter changes of lines) only effect Laplacian matrix $\mathfrak{L}$!

$$E = \begin{bmatrix} I_{n_g} & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_{\sigma(t)} = \begin{bmatrix} 0 & I_{n_g} & 0 \\ -Z^{-1} & -D & \begin{bmatrix} Z^{-1} & 0 \end{bmatrix} \\ \begin{bmatrix} Z^{-1} \\ 0 \end{bmatrix} & 0 & -\mathfrak{L}_{\sigma(t)} - \begin{bmatrix} Z^{-1} & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ I_{n_g} & 0 \\ 0 & I_{n_g+n_b} \end{bmatrix}$$

Simulation





Parameters:

$m_1 = m_2 = 1$

$z_1 = z_2 = 0.1$

$D_1 = D_2 = 0.01$

and Laplacian matrices for both modes:

$$\mathfrak{L}_1 = \begin{bmatrix} -0.01 & 0 & 0.005 & 0.005 \\ 0 & -5.005 & 0.005 & 5 \\ 0.005 & 0.005 & -0.02 & 0.01 \\ 0.005 & 5 & 0.01 & -5.015 \end{bmatrix}, \quad \mathfrak{L}_2 = \begin{bmatrix} -2.005 & 0 & 0.005 & 2 \\ 0 & -5.005 & 0.005 & 5 \\ 0.005 & 0.005 & -0.02 & 0.01 \\ 2 & 5 & 0.01 & -7.01 \end{bmatrix}$$

Modelling

Instability due to switching

Sufficient condition for stability under arbitrary switching

Stability and Lyapunov functions

$$E_\sigma \dot{x} = A_\sigma x \qquad \textbf{(swDAE)}$$

Theorem (cf. *Liberzon and T. 2012*)

*Assume (**swDAE**) to be regular and index one. If*

1. *each mode is stable with Lyapunov function* $V_p(\cdot)$
2. $V_q(\Pi_q x) \leq V_p(x)$ *for all* p, q *and all* $x \in \operatorname{im} \Pi_p$

*then (**swDAE**) is stable under arbitrary switching.*

D. Liberzon, S. Trenn: Switched nonlinear differential algebraic equations: Solution theory, Lyapunov functions, and stability. *Automatica*, 48 (5), pp. 954–963, 2012.

Remark

If E-matrix is switch-independent and has the form $E = \begin{bmatrix} E_1 & 0 \\ 0 & 0 \end{bmatrix}$ with invertible E_1, then $V_q(\Pi_q x) = V_q(x)$ for all $x \in \operatorname{im} \Pi_p$.

$\hookrightarrow$ common Lyapunov function guarantees stability

Key lemma

Lemma

Consider (E,A) with structure

$$E = \begin{bmatrix} E_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} A_1 & A_2 & 0 \\ A_3 & -\mathfrak{L}_1 + A_4 & -\mathfrak{L}_2 \\ 0 & -\mathfrak{L}_3 & -\mathfrak{L}_4 \end{bmatrix},$$

where $\mathfrak{L} = \begin{bmatrix} \mathfrak{L}_1 & \mathfrak{L}_2 \\ \mathfrak{L}_3 & \mathfrak{L}_4 \end{bmatrix}$ *is a (weighted) Laplacian matrix. If*

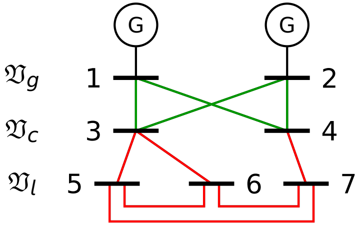
- (E, A) *is regular, index one and stable*
- $\operatorname{rank} \mathfrak{L}_3 = 1$

then $\exists$ *common Lyapunov function for all possible* $\mathfrak{L}_4$

Structural assumption for stability

Assume $\mathfrak{V} = \mathfrak{V}_g \dot\cup \mathfrak{V}_c \dot\cup \mathfrak{V}_l$ such that

1. $\mathfrak{V}_g$ are the generator busses
2. no edges between $\mathfrak{V}_g$ and $\mathfrak{V}_l$
3. full connection between $\mathfrak{V}_g$ and $\mathfrak{V}_c$
4. Laplacian of edges between $\mathfrak{V}_g$ and $\mathfrak{V}_c$ has rank one
5. topological changes only occur in edges in $\mathfrak{V}_c \cup \mathfrak{V}_l$



Theorem (Groß et al. 2018)

Under above assumptions, stability is preserved under arbitrary switching.

T.B. Gross, S. Trenn, A. Wirsen: Switch induced instabilities for stable power system DAE models. To appear in *Proceedings of IFAC Conference on Analysis and Design of Hybrid Systems (ADHS 2018)*, Oxford, UK, 2018.

Summary

- Presented a simple linear DAE-model for power grids
- This DAE model is regular, index 1 and stable
- Sudden repeated changes in line parameter may lead to instability
- Topological conditions are presented which prevent instability

Open questions

- Physical interpretation
- Nonlinear and more detailed model