Switch observability for a class of inhomogeneous switched DAEs

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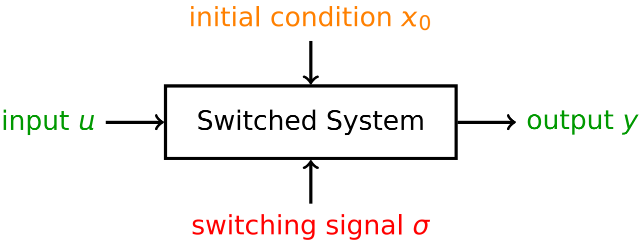
The observability problem



Observability questions

- Is there a unique x_0 for any given σ, u, y? → observability ✓
- Is there a unique (x_0, σ) for any given u and y?
  → (x, σ)-observability
- Is there a unique σ for any given u, y and unknown x_0?
  → σ-observability

(x,σ)-observability vs. σ-observability

First (surprising?) result for *linear* systems

$$(x,\sigma)\text{-observability} \quad \Longleftrightarrow \quad \sigma\text{-observability}$$

$\Longrightarrow$ is clear.
Main argument for $\Longleftarrow$:
Choose initial values $x_0^1 \neq x_0^2$ with the same input-output behavior
$\rightarrow x_0 := x_0^1 - x_0^2 \neq 0$ gives $y \equiv 0$
$\rightarrow y \equiv 0$ also results from $x_0 = 0$ and any σ

Corollary for *linear* systems

$$\sigma\text{-observability} \quad \Longrightarrow \quad \text{each individual mode observable}$$

Weaker observability notion

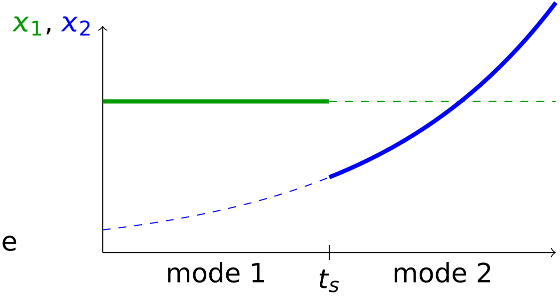
$$\dot{x} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} x$$

$$y = C_{\sigma} x$$

with

$$C_1 = [1, 0]$$
$$C_2 = [0, 1]$$

$\rightarrow$ not observable



Switch observability (σ_1-observability)

Recover x and σ from u and y, if at least one switch occurs

The simplest case

$$\dot{x} = A_\sigma x \qquad \mathcal{O}_k := \begin{bmatrix} C_k \\ C_k A_k \\ C_k A_k^2 \\ \vdots \end{bmatrix}$$
$$y = C_\sigma x$$

Theorem (cf. *Küsters & Trenn*, Automatica 2018)

$$\sigma\text{-observability} \iff \forall i \neq j:\ \operatorname{rank}[\mathcal{O}_i\ \mathcal{O}_j] = 2n$$

$$\sigma_1\text{-observability} \iff \forall i \neq j, p \neq q, (i,j) \neq (p,q):\ \operatorname{rank}\begin{bmatrix} \mathcal{O}_i & \mathcal{O}_p \\ \mathcal{O}_j & \mathcal{O}_q \end{bmatrix} = 2n$$

$$t_S\text{-observability} \iff \forall i \neq j:\ \operatorname{rank}[\mathcal{O}_i - \mathcal{O}_j] = n$$

Adding inputs

$$\dot{x} = A_\sigma x + B_\sigma u$$
$$y = C_\sigma x + D_\sigma u$$

Input-depending observability

$$\Sigma(A_\sigma, C_\sigma) \text{ } \sigma\text{-observable} \quad \not\Leftrightarrow \quad \Sigma(A_\sigma, B_\sigma, C_\sigma, D_\sigma) \text{ } \sigma\text{-observable}$$

Example:

$$\dot{x} = x \qquad \dot{x} = 0 + u$$
$$y = x \qquad y = x$$

is σ-observable but not distinguishable for $u(t) = e^t$ and $x(0) = 1$

Adding inputs

$$\dot{x} = A_\sigma x + B_\sigma u$$
$$y = C_\sigma x + D_\sigma u$$

Input-depending observability

$\Sigma(A_\sigma, C_\sigma)$ σ-observable $\not\Leftrightarrow$ $\Sigma(A_\sigma, B_\sigma, C_\sigma, D_\sigma)$ σ-observable

Strong vs. weak observability

observable for all u $\not\Leftrightarrow$ observable for some/almost all u

Further technicalities

Analytic vs. smooth inputs and equivalent switching signals

All problems resolvable → see our 2018 Automatica paper

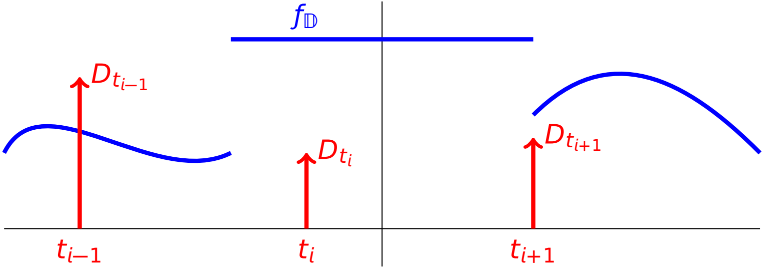
Adding algebraic constraints

$$E_\sigma \dot{x} = A_\sigma x$$
$$y = C_\sigma x$$

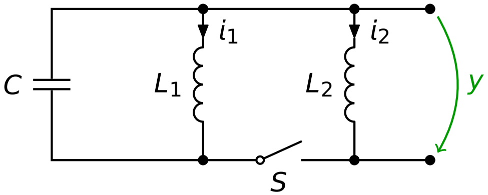
Extended solution space

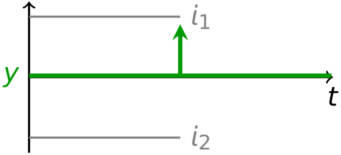
Distributional solution space $\longrightarrow$ Dirac impulses possible

Suitable solution space: Piecewise-smooth distrubutions



Impulses important for observability

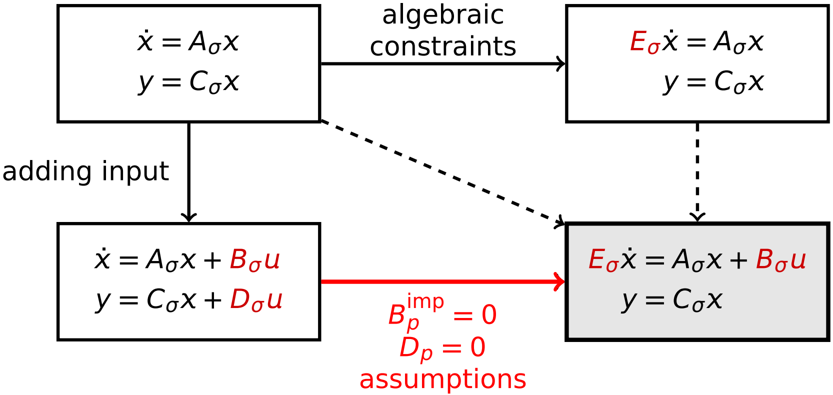




Switch		obsv.
open	$y \equiv 0$ for any internal states	✗
closed	equilibrium $i_1 = -i_2 = \text{const} \;\rightarrow\; y \equiv 0$	✗
closing	$y = 0$ jumps to $\neq 0$	✓
opening	non-equilibrium: $y \neq 0$ jumps to zero (+ Imp.)	✓
	equilibrium: $y(t) = 0\ \forall t$, but with impulse in y	✓

The switch-induced impulse is required to determine x and σ.

System classes



Solution formula for nonswitched DAEs

$$E_p\dot{x} = A_p x + B_p u, \quad x(0^-) = x_0$$

has unique solution on $(0, \infty)$

$$x(t) = e^{A_p^{\text{diff}} t}\Pi_p x_0 + \int_0^t e^{A_p^{\text{diff}}(t-s)} B_p^{\text{diff}} u(s)\ \mathrm{d}s - \sum_{i=0}^{n-1} (E_p^{\text{imp}})^i B_p^{\text{imp}} u^{(i)}(t)$$

Assumption: $B_p^{\text{imp}} = 0 \quad \rightarrow \quad$ DAE behaves like $\dot{x} = A_p^{\text{diff}} x + B_p^{\text{diff}} u$

Jumps and Dirac impulses still present at switches

$$x(t_p^+) = \Pi_p x(t_p^-)$$

$$x[t_p] = -\sum_{i=0}^{n-1} (E_p^{\text{imp}})^{i+1} x(t_p^-)\delta_{t_p}^{(i)}$$

Observability characterizations

$$\begin{aligned} E_\sigma x &= A_\sigma x + B_\sigma u \\ y &= C_\sigma x \end{aligned} \quad \text{regular with corresponding} \quad \begin{array}{l} \Pi_p, A_p^{\text{diff}}, B_p^{\text{diff}}, C_p^{\text{diff}}, \\ E_p^{\text{imp}}, B_p^{\text{imp}}, C_p^{\text{imp}} \end{array}$$

Notation:

$$\mathcal{O}_k = \begin{bmatrix} C_k^{\text{diff}} \\ C_k^{\text{diff}} A_k^{\text{diff}} \\ C_k^{\text{diff}} {A_k^{\text{diff}}}^2 \\ \vdots \end{bmatrix}, \quad \boldsymbol{O}_k = \begin{bmatrix} C_k^{\text{imp}} E_k^{\text{imp}} \\ C_k^{\text{imp}} {E_k^{\text{imp}}}^2 \\ C_k^{\text{imp}} {E_k^{\text{imp}}}^3 \\ \vdots \end{bmatrix}, \quad \Gamma_k = \begin{bmatrix} 0 & & & \\ C_k^{\text{diff}} B_k^{\text{diff}} & 0 & & \\ C_k^{\text{diff}} A_k^{\text{diff}} B_k^{\text{diff}} & C_k^{\text{diff}} B_k^{\text{diff}} & \ddots & \\ C_k^{\text{diff}} {A_k^{\text{diff}}}^2 B_k^{\text{diff}} & C_k^{\text{diff}} A_k^{\text{diff}} B_k^{\text{diff}} & \ddots & \ddots \\ \vdots & \vdots & & \ddots & \ddots \end{bmatrix}$$

Observability characterizations

$$\begin{aligned} E_\sigma \dot{x} &= A_\sigma x + B_\sigma u \\ y &= C_\sigma x \end{aligned}$$

regular with corresponding $\Pi_p, A_p^{\text{diff}}, B_p^{\text{diff}}, C_p^{\text{diff}}, E_p^{\text{imp}}, B_p^{\text{imp}}, C_p^{\text{imp}}$

Theorem (Assumption $B^{\text{imp}} = 0$)

σ-observability $\iff$

$$\operatorname{rank}\begin{bmatrix} \mathcal{O}_i & \mathcal{O}_j & \Gamma_i - \Gamma_j \end{bmatrix} = \operatorname{rank}\Pi_i + \operatorname{rank}\Pi_j + \operatorname{rank}(\Gamma_i - \Gamma_j)$$

+ *technical impulse condition*

Observability characterizations

$$\begin{aligned} E_\sigma x &= A_\sigma x + B_\sigma u \\ y &= C_\sigma x \end{aligned}$$

regular with corresponding $\Pi_p, A_p^{\text{diff}}, B_p^{\text{diff}}, C_p^{\text{diff}}, E_p^{\text{imp}}, B_p^{\text{imp}}, C_p^{\text{imp}}$

Theorem (Assumption $B^{\text{imp}} = 0$)

σ_1-observability $\iff$

t_S-observability +

$$\operatorname{rank}\begin{bmatrix} \mathcal{O}_i & \mathcal{O}_p & \Gamma_i - \Gamma_p \\ \mathcal{O}_j \Pi_i & \mathcal{O}_q \Pi_p & \Gamma_j - \Gamma_q \\ \mathbf{O}_j \Pi_i & \mathbf{O}_q \Pi_p & 0 \end{bmatrix} = \operatorname{rank} \Pi_i + \operatorname{rank} \Pi_p + \operatorname{rank}\begin{bmatrix} \Gamma_i - \Gamma_p \\ \Gamma_j - \Gamma_q \end{bmatrix} - \dim \mathcal{M}_{i,j,p,q}$$

where $\mathcal{M}_{i,j,i,q} = \operatorname{im} \Pi_i \cap \ker E_j \cap \ker E_q$ *and* $\mathcal{M}_{i,j,p,q} = \{0\}$ *for* $i \neq p$