university of groningen
faculty of science and engineering
johann bernoulli institute for mathematics and computer science

Switch-observer for switched linear systems

Stephan Trenn

Jan C. Willems Center for Systems and Control
University of Groningen, Netherlands

Joint work with **Ferdinand Küsters** and **Andreas Wirsen**, Fraunhofer ITWM, Kaiserslautern, Germany

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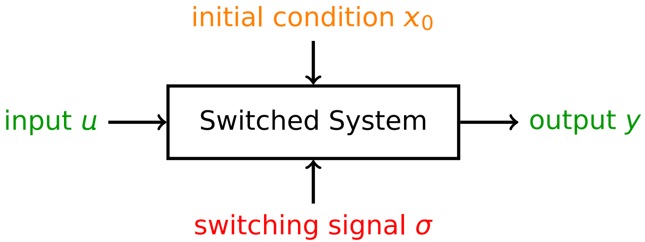
The observability problem



Observability questions

- Is there a unique x_0 for any given σ, u, y? $\to$ observability ✓
- Is there a unique (x_0, σ) for any given u and y?
  $\to$ (x, σ)-observability
- Is there a unique σ for any given u, y and unknown x_0?
  $\to$ σ-observability $\iff$ (x, σ)-observability

“Trivial” observer design for (x,σ)-obs.

Instantenous observability

(x,σ)-observability $\implies$ local state and mode observability

Observer design

1. For each mode run a classical state observer
2. Pick the one which converges $\rightarrow$ mode and state estimation
3. Repeat

Nothing switch specific

Information at the switch (e.g. jumps) not utilized.

Weaker observability notion

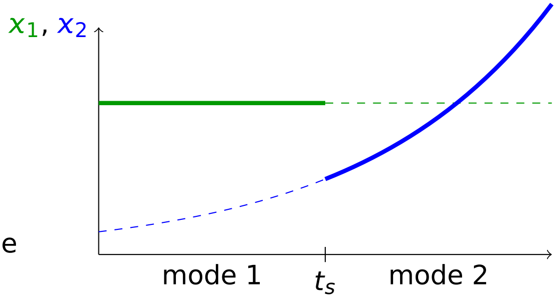
$$\dot{x} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$y = C_{\sigma} x$$

with

$$C_1 = [1, 0]$$
$$C_2 = [0, 1]$$

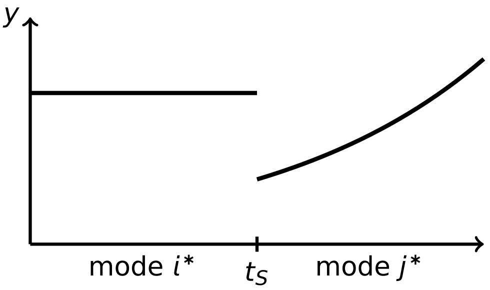
$\rightarrow$ not observable



Switch observability (σ_1-observability)

Recover x and σ from u and y, if at least one switch occurs

Overall observer design



(0. Detect switching time t_S.)
1a. Run partial state observers on $(t_S - \tau, t_S)$ for all modes.
1b. Run partial state observers on $(t_S, t_S + \tau)$ for all modes.
2. Combine partial information to find (i^*, j^*) and state estimation $\widehat{x}(t_S)$

Partial state observer

$$\dot{x} = A_p \dot{x} + B_p u, \qquad \mathcal{O}_p := \begin{bmatrix} C_p \\ C_p A_p \\ \vdots \\ C_p A_p^{n-1} \end{bmatrix} \qquad r_p := \operatorname{rank} \mathcal{O}_p$$
$$y = C_p x + D_p u,'$$

Choose orthogonal $Z_p \in \mathbb{R}^{n \times r_p}$ with $\operatorname{im} Z_p = \operatorname{im} \mathcal{O}_p^\top$, then

$$\dot{z}_p = Z_p{}^\top A_p Z_p z_p + Z_p^\top B_p u$$
$$y = C_p Z_p z_p + D_p u$$

is observable

Definition (Partial state observer)

Any observer for $z_p = Z_p^\top x$ is a partial state observer.

Mode dependence

Z_p and size r_p are mode dependent.

Reasonable modes

Definition (Reasonable modes)

Mode i is reasonable on $(t_S - \tau, t_S)$ $\quad :\Leftrightarrow$

$$\exists x_i^{t_S} : y = C_i x_i + D_i u \quad \text{where } \dot{x}_i = A_i x_i + B_i u,\ x_i(t_S) = x_i^{t_S}$$

In particular, i^* is reasonable on $(t_S - \tau, t_S)$.

Crucial property of reasonable modes

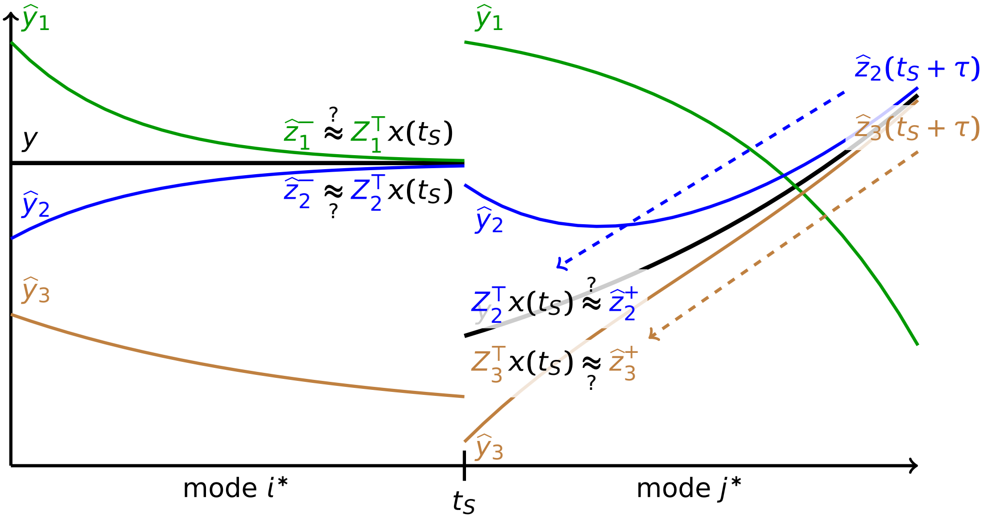
Partial state observers “converge” for all reasonable modes, i.e.

$$y \approx C_i Z_i \widehat{z}_i + D_i u \quad \text{on } (t_S - \varepsilon, t_S) \quad \forall \text{ reasonable } i$$

Analog definition for reasonable modes j on $(t_S, t_S + \tau)$, with

$$y \approx C_j Z_j \widehat{z}_j + D_j u \quad \text{on } (t_S + \tau - \varepsilon, t_S + \tau) \quad \forall \text{ reasonable } j$$

Illustration of Steps 1 and 2



Combining partial state estimations

Question

How to combine the obtained information before and after the switch?

Obvious fact

(x,σ_1)-observability $\implies$ observability for known σ with one switch
$\implies$ $\ker \mathcal{O}_i \cap \ker \mathcal{O}_j = \{0\} \quad \forall i \neq j$
$\implies$ $\operatorname{rank} [Z_i, Z_j] = n \quad \forall i \neq j$

State estimation candidates

For $(i,j) = (i^*, j^*)$ we have

$$\begin{pmatrix} \widehat{z}_i^- \\ \widehat{z}_j^+ \end{pmatrix} \approx \begin{bmatrix} Z_i^\top \\ Z_j^\top \end{bmatrix} x(t_S) \implies x(t_S) \approx \begin{bmatrix} Z_i^\top \\ Z_j^\top \end{bmatrix}^\dagger \begin{pmatrix} \widehat{z}_i^- \\ \widehat{z}_j^+ \end{pmatrix} =: \widehat{x}_{ij}$$

Final step

Theorem

For sufficiently accurate partial observers and for all reasonable (i,j)

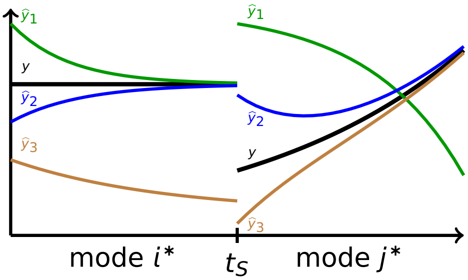
$$(i,j) = (i^*, j^*) \quad \Longrightarrow \quad \begin{bmatrix} Z_i^\top \\ Z_j^\top \end{bmatrix} \widehat{x}_{ij} \approx \begin{bmatrix} \widehat{z}_i^- \\ \widehat{z}_j^+ \end{bmatrix}$$

$$(i,j) \neq (i^*, j^*) \quad \Longrightarrow \quad \begin{bmatrix} Z_i^\top \\ Z_j^\top \end{bmatrix} \widehat{x}_{ij} \not\approx \begin{bmatrix} \widehat{z}_i^- \\ \widehat{z}_j^+ \end{bmatrix}$$

Summary: Observer design

1. Run partial state observers for

$$\dot{z}_p = Z_p^\top A_p Z_p z_p + Z_p^\top B_p u$$
$$y = C_p Z_p z_p + D_p u$$
$$\operatorname{im} Z_p := \operatorname{im} \mathcal{O}_p^\top$$



on $(t_S - \tau, t_S)$ and $(t_S, t_S + \tau)$

$\hookrightarrow$ reasonable mode pairs (i, j) and partial state estimates $\widehat{z}_i^-, \widehat{z}_j^+$

2. Calculate candidate full state estimations $\widehat{x}_{ij} = \begin{bmatrix} Z_i^\top \\ Z_j^\top \end{bmatrix}^\dagger \begin{pmatrix} \widehat{z}_i^- \\ \widehat{z}_j^+ \end{pmatrix}$

$\hookrightarrow$ choose unique $(i, j) = (i^*, j^*)$ for which $\begin{bmatrix} Z_i^\top \\ Z_j^\top \end{bmatrix} \widehat{x}_{ij} \approx \begin{bmatrix} \widehat{z}_i^- \\ \widehat{z}_j^+ \end{bmatrix}$

Discussion and Extensions

› Novel observability notion: *Switch observability*
- Assumes that at least one switch occurs
- Individual modes can have unobservable states
- Better suited for fault-induced switches

› Novel observer-design suited to switch-observable systems
- Based on partial state observers
- Linear costs in number of possible faults
- Extension to switched DAEs possible

- Knowledge of switching time assumed
- Unavoidable time delay until result is available
- Choice of specific left-inverse of $[Z_i, Z_j]^\top$ crucial for performance