Switch.Observer
for switched linear systems

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The observability problem

Observability notions

Observer design

Summary

Switched System

initial condition $x_0$

input $u$ → output $y$

switching signal $\sigma$

Observability questions

› Is there a unique $x_0$ for any given $\sigma$, $u$, $y$? → observability ✓
› Is there a unique $(x_0, \sigma)$ for any given $u$ and $y$?
  → $(x, \sigma)$-observability
› Is there a unique $\sigma$ for any given $u$, $y$ and unknown $x_0$?
  → $\sigma$-observability $\iff (x, \sigma)$-observability
“Trivial” observer design for \((x, \sigma)\)-obs.

<table>
<thead>
<tr>
<th>Instantenous observability</th>
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<tbody>
<tr>
<td>((x, \sigma))-observability \implies\ local state and mode observability</td>
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<tr>
<th>Observer design</th>
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<tbody>
<tr>
<td>1. For each mode run a classical state observer</td>
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<td>2. Pick the one which converges (\rightarrow) mode and state estimation</td>
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<td>3. Repeat</td>
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<tr>
<th>Nothing switch specific</th>
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<td>Information at the switch (e.g. jumps) not utilized.</td>
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Weaker observability notion

\[ \dot{x} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \]

\[ y = C_0 x \]

with

\[ C_1 = [1, 0] \]
\[ C_2 = [0, 1] \]

\[ \tilde{y} = C_\sigma \]

\[ \text{not observable} \]

Switch observability (\(\sigma_1\)-observability)

Recover \(x\) and \(\sigma\) from \(u\) and \(y\), if at least one switch occurs
Overall observer design

(0. Detect switching time $t_S$.)

1a. Run partial state observers on $(t_S - \tau, t_S)$ for all modes.
1b. Run partial state observers on $(t_S, t_S + \tau)$ for all modes.

2. Combine partial information to find $(i^*, j^*)$ and state estimation $\hat{x}(t_S)$
Partial state observer

\[
\begin{align*}
\dot{x} &= A_p \dot{x} + B_p u, \\
y &= C_p x + D_p u,
\end{align*}
\]

\[
O_p := \begin{bmatrix}
C_p \\
C_p A_p \\
\vdots \\
C_p A_p^{n-1}
\end{bmatrix}
\]

\[
r_p := \text{rank } O_p
\]

Choose orthogonal \( Z_p \in \mathbb{R}^{n \times r_p} \) with \( \text{im } Z_p = \text{im } O_p^T \), then

\[
\begin{align*}
\dot{z}_p &= Z_p^T A_p Z_p z_p + Z_p^T B_p u \\
y &= C_p Z_p z_p + D_p u
\end{align*}
\]

is observable

**Definition (Partial state observer)**

Any observer for \( z_p = Z_p^T x \) is a **partial state observer**.

**Mode dependence**

\( Z_p \) and size \( r_p \) are mode dependent.
Reasonable modes

Definition (Reasonable modes)
Mode $i$ is **reasonable** on $(t_S - \tau, t_S)$ :⇔

$$
\exists x_i^{t_S} : y = C_i x_i + D_i u \quad \text{where} \quad \dot{x}_i = A_i x_i + B_i u, \quad x_i(t_S) = x_i^{t_S}
$$

In particular, $i^*$ is reasonable on $(t_S - \tau, t_S)$.

Crucial property of reasonable modes
Partial state observers “converge” for all reasonable modes, i.e.

$$
y \approx C_i \hat{Z}_i + D_i u \quad \text{on} \quad (t_S - \varepsilon, t_S) \quad \forall \text{reasonable } i
$$

Analog definition for reasonable modes $j$ on $(t_S, t_S + \tau)$, with

$$
y \approx C_j \hat{Z}_j + D_j u \quad \text{on} \quad (t_S + \tau - \varepsilon, t_S + \tau) \quad \forall \text{reasonable } j
$$
Illustration of Steps 1 and 2

\[ \hat{y}_1 \]
\[ \hat{y}_2 \]
\[ \hat{y}_3 \]

\[ Z_1 \approx Z_1^T x(t_S) \]
\[ Z_2 \approx Z_2^T x(t_S) \]

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Combining partial state estimations

Question
How to combine the obtained information before and after the switch?

Obvious fact

\((x, \sigma_1)\)-observability \implies \text{observability for known } \sigma \text{ with one switch}
\implies \ker \mathcal{O}_i \cap \ker \mathcal{O}_j = \{0\} \quad \forall i \neq j
\implies \text{rank } [Z_i, Z_j] = n \quad \forall i \neq j

State estimation candidates

For \((i, j) = (i^*, j^*)\) we have

\[
\begin{bmatrix}
\hat{Z}_i^- \\
\hat{Z}_j^+
\end{bmatrix}
\approx
\begin{bmatrix}
Z_i^T \\
Z_j^T
\end{bmatrix} x(t_S) \implies x(t_S) \approx
\begin{bmatrix}
Z_i^T \\
Z_j^T
\end{bmatrix}^+ \begin{bmatrix}
\hat{Z}_i^- \\
\hat{Z}_j^+
\end{bmatrix} =: \hat{x}_{ij}
\]
Final step

Theorem

For sufficiently accurate partial observers and for all reasonable \((i, j)\)

\[
(i, j) = (i^*, j^*) \implies \begin{bmatrix} Z_i^T \\ Z_j^T \end{bmatrix} \hat{x}_{ij} \approx \begin{bmatrix} \hat{z}_i^- \\ \hat{z}_j^+ \end{bmatrix}
\]

\[
(i, j) \neq (i^*, j^*) \implies \begin{bmatrix} Z_i^T \\ Z_j^T \end{bmatrix} \hat{x}_{ij} \not\approx \begin{bmatrix} \hat{z}_i^- \\ \hat{z}_j^+ \end{bmatrix}
\]
Summary: Observer design

1. Run partial state observers for
\[
\dot{z}_p = Z_p^T A_p z_p + Z_p^T B_p u \\
y = C_p Z_p z_p + D_p u \\
im Z_p := im \mathcal{O}_p^T
\]
on \((t_S - \tau, t_S)\) and \((t_S, t_S + \tau)\)

\[\rightarrow\] reasonable mode pairs \((i, j)\) and partial state estimates \(\hat{Z}_i, \hat{Z}_j^+\)

2. Calculate candidate full state estimations \(\hat{x}_{ij} = \begin{bmatrix} z_i^T \\ z_j^T \end{bmatrix}^T \begin{bmatrix} \hat{Z}_i^- \\ \hat{Z}_j^+ \end{bmatrix}\)

\[\rightarrow\] choose unique \((i, j) = (i^*, j^*)\) for which
\[
\begin{bmatrix} z_i^T \\ z_j^T \end{bmatrix} \hat{x}_{ij} \approx \begin{bmatrix} \hat{Z}_i^- \\ \hat{Z}_j^+ \end{bmatrix}
\]
Discussion and Extensions

› Novel observability notion: *Switch observability*
  - Assumes that at least one switch occurs
  - Individual modes can have unobservable states
  - Better suited for fault-induced switches

› Novel observer-design suited to switch-observable systems
  - Based on partial state observers
  - Linear costs in number of possible faults
  - Extension to switched DAEs possible
  - Knowledge of switching time assumed
  - Unavoidable time delay until result is available
  - Choice of specific left-inverse of $[Z_i, Z_j]^T$ crucial for performance