Indiscernible topological variations in DAE networks

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Abstract

A problem of characterizing conditions under which a topological change in a network of differential algebraic equations (DAEs) can go undetected is considered. It is shown that initial conditions for which topological changes are indiscernible belong to a generalized eigenspace shared by the nominal system and the system resulting from a topological change. A condition in terms of eigenvectors of the nominal system is derived to check for existence of possibly indiscernible topological changes. For homogenous networks this conditions simplifies to existence of an eigenvector of the Laplacian of network having equal components. Lastly, a rank condition is derived which can be used to check if a topological change preserves regularity of the nominal network.

Keywords: Differential Algebraic Equations (DAEs), DAE networks, Time-varying topologies

1. Introduction

Control theory of dynamical networks and multiagent systems has gained enormous popularity in the last years because it involves numerous important applications, as well as many unsolved mathematical questions. In the engineering domain, dynamical networks and multiagent systems networks naturally arise in cooperative robotics, surveillance and environment monitoring Ögren et al. (2004); Beard et al. (2006); Arcak (2007), as well as man-made infrastructures such as electrical power grids Chakrabortty and Khargonekar (2013) and transportation networks Banavar et al. (2000).

Networks can be modelled in terms of a graph, where the nodes represent the various network agents and the edges represent the interaction among the nodes. The overall network dynamics is then the result of the dynamics of each node and the network topology (the interconnection structure formed by the edges). In this context, a key question is whether a *topological change* in the form of a removal or addition of an edge may have an effect on the overall dynamics. This question is relevant for both analysis and design purposes. In terms of analysis, it is important to understand whether a topological change cannot be detected because this indicates the existence of an event (for example the loss of a transmission line in a power grid) which cannot be detected. Also, understanding which topological changes may give rise to drastically different dynamics is key to identify critical links in the networks. This question is also relevant from a design perspective since it helps to understand how one can possibly modify the network structure without altering too much the network behavior.

The problem of detecting network topological changes has therefore attracted significant attention in the last decade; see for instance Barooah (2008); Rahimian et al. (2012); Dhal et al. (2013); Rahimian and Preciado (2015); Torres et al. (2015); Battistelli and Tesi (2015, 2017); Costanzo et al. (2017). In all the aforementioned works, however, the analysis is confined to networks whose dynamics can be fully described in terms of differential equations. In contrast, there are no results dealing with dynamics that obey differential-algebraic equations (DAEs), apart from our own preliminary conference publication Küsters et al. (2017) which only considers the SISO case and also does not characterize the set of indiscernible initial states. Networks of DAEs arise in several applications of practical interest, examples being water distribution and electrical networks, where the algebraic equations describe conservation laws related to mass and energy balance.

In this paper, we consider networks of DAEs with diffusive coupling, and study under what conditions topological changes (in particular, a removal or addition of an edge) cannot be detected from observations of the network dynamics, referring to this event as "*indiscernibility*". We approach this problem from the perspective of control theory and provide necessary and sufficient conditions for indiscernibility that depend on the common eigenspaces of the nominal (before the addition/removal of an edge) and modified network configuration. In this respect, a very interesting result is that indiscernibility can be checked by only looking at the eigenspace of the nominal network configuration. In many practical cases, the latter is usually known in advance since it represents the configuration with which the network is designed to operate. This renders the approach appealing from a practical viewpoint since it allows one to check the existence of indiscernible topological changes with no need to look at all the possible modified topologies. Another interesting result is that the considered approach is general enough so as to include the case where each network node obeys different dynamics, and has possibly different state dimension.

The results presented here consider discernibility based on the whole state trajectory. This is just a preliminary step, because once a topological change results in a change of the dynamics, the next question is, whether this change can actually be seen by a limited amount of sensors in the network. This problem has been widely studied in the general framework of switched systems Vidal et al. (2002); Babaali and Egerstedt (2004); Baglietto et al. (2007); De Santis (2011); Baglietto et al. (2014); however, these results do not take into account the special structure of topological changes and it is a topic of future research to consider discernibility also for networks with a limited amount of measurements.

Finally we would like to note that detecting topological changes can be seen as part of the more general problem of network reconstruction/identification Timme (2007); Chowdhary et al. (2011); Sanandaji et al. (2011); Nabavi and Chakrabortty (2016); Angulo et al. (2017). In fact, checking whether or not two different network configurations can generate the same dynamics can also be approached by asking under what conditions one can uniquely identify from observations the coupling parameters of the network. However, the problem considered here has much more "structure" than a generic topology identification problem. For example, identification approaches do not assume prior knowledge of a nominal network configuration. In the present context, this knowledge makes it possible to provide conditions on discernibility that can be checked by only looking at the properties of the nominal network configuration.

This paper is organized as follows. First, we define a nominal network of DAEs and obtain a resulting overall DAE. We also note the effect of addition/removal of an edge on the overall DAE and characterize it as a rank one update to system matrix. Then, we introduce the notion of indiscernibility and bring out a connection between indiscernibility and existence of common generalized eigenspace. This leads to a simple condition on nominal network which can be used to characterize all topological changes which are possibly-indiscernible. Afterwards, we consider a special case of homogeneous networks and obtain a condition for possibly indiscernible topological change which can be checked solely from eigenvectors of the Laplacian of nominal network. Lastly, we give a simple rank condition which helps us check whether a topological change is regularity preserving.

2. System class

We consider a family of $N \in \mathbb{N}$ differential algebraic equations (DAEs),

$$\begin{aligned} E_i \dot{x}_i &= A_i x_i + B_i u_i, \\ y_i &= C_i x_i, \end{aligned} \qquad i \in \{1, 2, \dots, N\}, \tag{1}$$

where $E_i, A_i \in \mathbb{R}^{n_i \times n_i}$, $n_i \in \mathbb{N}$, $B_i, C_i^\top \in \mathbb{R}^{n_i \times p}$, $p \in \mathbb{N}$. Note that each system can have its own state dimension and we allow multiple inputs and outputs (but with the same number p for all systems).

The systems are coupled with each other via diffusive coupling, i.e. for a given undirected coupling graph $\mathfrak{G} = (\mathfrak{V}, \mathfrak{E})$ with vertices $\mathfrak{V} = \{1, 2, \dots, N\}$ and edges $\mathfrak{E} \subseteq \mathfrak{V} \times \mathfrak{V}$ the input of the i-th system is determined by the output of all neighbouring systems as follows:

$$u_i = \sum_{k:(i,k)\in\mathfrak{E}} w_{ik}(y_k - y_i), \tag{2}$$

where $w_{ij} > 0$ with $w_{ji} = w_{ij}$ for $i, j \in \mathfrak{V}$.

Let the weighted Laplacian matrix $\mathfrak{L} = [\ell_{i,j}]_{i,j\in\mathfrak{V}}$ of the graph $\mathfrak{G}$ be given by

$$\ell_{ij} = \begin{cases} -w_{ij}, & i \neq j,\ (i,j) \in \mathfrak{E}, \\ 0, & i \neq j,\ (i,j) \notin \mathfrak{E}, \\ \sum\limits_{k:(i,k)\in\mathfrak{E}} w_{ik}, & i = j; \end{cases} \tag{3}$$

note that $\mathfrak{L} \in \mathbb{R}^{N\times N}$ is a symmetric and positive semidefinite matrix. We then can write the coupled dynamics in compact form as

$$\mathcal{E}\dot{x} = (\mathcal{A} - \mathcal{B}(\mathfrak{L} \otimes I_p)\mathcal{C})x =: \mathcal{A}_{\mathfrak{L}} x, \tag{4}$$

where, for $n := \sum_{i=1}^N n_i$,

$$\begin{aligned} x &:= (x_1^\top, x_2^\top, \dots, x_N^\top)^\top, \\ \mathcal{E} &:= \operatorname{diag}\{E_1, \dots, E_N\} \in \mathbb{R}^{n\times n}, \\ \mathcal{A} &:= \operatorname{diag}\{A_1, \dots, A_N\} \in \mathbb{R}^{n\times n}, \\ \mathcal{B} &:= \operatorname{diag}\{B_1, \dots, B_N\} \in \mathbb{R}^{n\times Np}, \\ \mathcal{C} &:= \operatorname{diag}\{C_1, \dots, C_N\} \in \mathbb{R}^{Np\times n}, \end{aligned}$$

and $\mathfrak{L}\otimes I_p \in \mathbb{R}^{Np\times Np}$ denotes the usual Kronecker product of $\mathfrak{L} \in \mathbb{R}^{N\times N}$ with the identity matrix $I_p \in \mathbb{R}^{p\times p}$.

3. Indiscernible initial states

In the following we are interested in the effect of a topological change in the coupling structure and its effect on the systems dynamics. In particular, we are interested in characterizing topological change which do *not* result in changes in the dynamics (for certain initial values). A topological change in the form of a removal/addition of an edge or, more general, a change in the edge weight, results in a change of the description (4) where $\mathfrak{L}$ is replaced by the new Laplacian matrix $\overline{\mathfrak{L}}$ while all other matrices $(\mathcal{E}, \mathcal{A}, \mathcal{B}, \mathcal{C})$ remain unchanged.

Definition 1 (Indiscernible initial states). *Consider the coupled system* (4)*. An initial value $x_0 \in \mathbb{R}^n$ is called* indiscernible *with respect to the topological change $\mathfrak{L} \to \overline{\mathfrak{L}}$ iff for all solutions x of $\mathcal{E}\dot{x} = \mathcal{A}_{\mathfrak{L}}x$ and all solutions $\overline{x}$ of $\mathcal{E}\dot{\overline{x}} = \mathcal{A}_{\overline{\mathfrak{L}}}\overline{x}$ the following implication holds:*

$$x(0) = x_0 = \overline{x}(0) \implies x(t) = \overline{x}(t) \quad \forall t \in \mathbb{R}.$$

Note that $x_0 = 0$ is always an indiscernible initial state (independently of the specific topological variation) and for certain topological variations it may be the only indiscernible initial state. We are now interested in fully characterizing the set of all indiscernible initial states. Towards this goal we will need to recall some basic properties about eigenvectors of matrix pairs, c.f. (Berger et al., 2012, Defs. 3.1&3.3).

Definition 2 (Eigenvalues and eigenvector chains). *For a matrix pair $(E, A) \in \mathbb{R}^{n\times n} \times \mathbb{R}^{n\times n}$ a complex number $\lambda \in \mathbb{C}$ is called* (generalized) eigenvalue *if there exists a nontrivial $v \in \mathbb{C}^n \setminus \{0\}$ such that $(A - \lambda E)v_1 = 0$. The set of all eigenvalues of (E, A) is denoted by* $\operatorname{spec}(E, A)$.

A tuple of (complex) vectors $(v_1, v_2, \dots, v_k) \in (\mathbb{C}\setminus\{0\})^k$ is called eigenvector chain (EVC) of (E, A) *for an eigenvalue $\lambda \in \mathbb{C}$ iff v_1 is an eigenvector and for all $i = 2, 3, \dots, k$:*

$$(A - \lambda E)v_i = Ev_{i-1}. \tag{5}$$

The eigenspace of order k *for eigenvalue $\lambda \in \mathbb{C}$ is recursively given by $\mathcal{V}_\lambda^0 := \{0\}$ and*

$$\mathcal{V}_\lambda^k := (A - \lambda E)^{-1}(E\mathcal{V}_\lambda^{k-1}) \subseteq \mathbb{C}^n;$$

here $(A-\lambda E)^{-1}$ stands for the set-valued preimage ($A-\lambda E$ is not invertible).

The limit $\mathcal{V}_\lambda := \bigcup_{k\in\mathbb{N}} \mathcal{V}_\lambda^k$ of the increasing subspace sequence $\mathcal{V}_\lambda^k$ is called generalized eigenspace *for eigenvalue λ; note that $\mathcal{V}_\lambda^1$ is the* space of eigenvectors *corresponding to λ.*

When introducing eigenvalues, eigenvectors and eigenvector chains, it is common to assume *regularity* of the matrix pair (E, A), i.e. the polynomial $\det(sE - A)$ is not identically zero. While this is not strictly necessary, most of the following properties only hold under the regularity assumption and we will mention this additional assumption appropriately.

An interesting characterization for eigenvector chains of a regular matrix pair (E, A) is the following (Berger et al., 2012, Prop. 3.8): $(v_1, v_2, \dots, v_k)$ is an eigenvector chain for eigenvalue $\lambda \in \mathbb{C}$ if, and only if, all (complex-valued) functions, $i = 1, 2, \dots, k$,

$$x_i(t) = e^{\lambda t}[v_1, v_2, \dots, v_i]\left(\frac{t^{i-1}}{(i-1)!}, \dots, \frac{t^2}{2}, t, 1\right)^\top \tag{6}$$

are linearly independent solutions of $E\dot{x} = Ax$; note that $x_i(0) = v_i$. In fact, the following stronger result holds (which is a simple consequence from the above characterization together with (Berger et al., 2012, Thm 3.6)):

Lemma 3. *For a regular matrix pair (E, A) with distinct eigenvalues $\{\lambda_1, \lambda_2, \dots, \lambda_d\} \in \mathbb{C}$ there exists for each $\ell \in \{1, 2, \dots, d\}$ and for each $j \in \{1, 2, \dots, \dim \mathcal{V}_{\lambda_\ell}^1\}$ a number $k_{\ell,j}$ and an eigenvector chain $(v_1^{\ell,j}, v_2^{\ell,j}, \dots, v_{k_{\ell,j}}^{\ell,j})$ for eigenvalue λ_ℓ such that* all *solutions of $E\dot{x} = Ax$ are given by*

$$x(t) = \sum_{\ell=1}^{d} e^{\lambda_\ell t} \sum_{j=1}^{\dim \mathcal{V}_{\lambda_\ell}^1} \sum_{i=1}^{k_{\ell,j}} \alpha_{\ell,j,i} \sum_{\eta=1}^{i} v_\eta^{\ell,j} \frac{t^{i-\eta}}{(i-\eta)!} \tag{7}$$

and the coefficients $\alpha_{\ell,j,i}$ are uniquely *determined by the initial value $x(0)$. In particular, the set*

$$\left\{ v_i^{\ell,j} \;\middle|\; \begin{array}{l} \ell \in \{1, \dots, d\}, \\ j \in \{1, \dots, \dim \mathcal{V}_{\lambda_\ell}^1\}, \\ i \in \{1, \dots, k_{\ell,j}\} \end{array} \right\}$$

is linearly independent and the coefficients $\alpha_{\ell,j,i}$ are determined by the unique decomposition

$$x(0) = \sum_{\ell=1}^{d} \sum_{j=1}^{\dim \mathcal{V}_{\lambda_\ell}^1} \sum_{i=1}^{k_{\ell,j}} \alpha_{\ell,j,i}\, v_i^{\ell,j}.$$

With the help of *common* EVCs it is now possible to characterize all indiscernible initial states as follows:

Theorem 4. *Consider a network with dynamics given by* (4) *and a regularity preserving*[1] *topological change $\mathfrak{L} \to \overline{\mathfrak{L}}$. Let*

$$\mathfrak{C}_{\mathfrak{L},\overline{\mathfrak{L}}} := \left\{ v \in \mathbb{C}^n \;\middle|\; \begin{array}{l} \exists (v_1, v_2, \dots, v_k) \text{ common EVC of} \\ (\mathcal{E}, \mathcal{A}_{\mathfrak{L}}), (\mathcal{E}, \mathcal{A}_{\overline{\mathfrak{L}}}) \text{ for the same} \\ \text{eigenvalue } \lambda \in \mathbb{C} \text{ and} \\ v = v_i \text{ for some } i \in \{1, 2, \dots, k\} \end{array} \right\}$$

be the set of all vectors which appear in a common eigenvector chain of $(\mathcal{E}, \mathcal{A}_{\mathfrak{L}})$ and $(\mathcal{E}, \mathcal{A}_{\overline{\mathfrak{L}}})$. Then $x_0 \in \mathbb{R}^n$ is an indiscernible initial state for the topological change $\mathfrak{L} \to \overline{\mathfrak{L}}$ if, and only if, it is in the span of all common eigenvector chains of $(\mathcal{E}, \mathcal{A}_{\mathfrak{L}})$ and $(\mathcal{E}, \mathcal{A}_{\overline{\mathfrak{L}}})$, i.e.

$$x_0 \in \operatorname{span} \mathfrak{C}_{\mathfrak{L},\overline{\mathfrak{L}}} \cap \mathbb{R}^n.$$

Proof. Sufficiency is easily seen by considering a linear combination of (common) solutions of the form (6).

For showing the converse implication, let us assume that x_0 is indiscernible i.e., $x \equiv \bar{x}$, where x denotes the solution of $\mathcal{E}\dot{x} = \mathcal{A}_{\mathfrak{L}}x$, $x(0) = x_0$ and $\bar{x}$ is the solution of $\mathcal{E}\dot{\bar{x}} = \mathcal{A}_{\mathfrak{L}}\bar{x}$, $\bar{x}(0) = x_0$; in particular, x is given by (7), where $(v_1^{\ell,j}, \dots, v_{k_{\ell,j}}^{\ell,j})$ is the j-th eigenvector chain of $(\mathcal{E}, \mathcal{A}_{\mathfrak{L}})$ for eigenvalue λ_ℓ and

$$\bar{x}(t) = \sum_{\ell=1}^{\bar{d}} e^{\bar{\lambda}_\ell t} \sum_{j=1}^{\dim \bar{\mathcal{V}}_{\bar{\lambda}_\ell}^1} \sum_{i=1}^{\bar{k}_{\ell,j}} \bar{\alpha}_{\ell,j,i} \sum_{\eta=1}^{i} \bar{v}_\eta^{\ell,j} \frac{t^{i-\eta}}{(i-\eta)!},$$

[1] i.e. the matrix pairs $(\mathcal{E}, \mathcal{A}_{\mathfrak{L}})$, $(\mathcal{E}, \mathcal{A}_{\overline{\mathfrak{L}}})$ are both regular

where $(\bar{v}_1^{\ell,j}, \ldots, \bar{v}_{k_{\ell,j}}^{\ell,j})$ is an eigenvector chain of $(\mathcal{E}, \mathcal{A}_{\overline{\mathfrak{L}}})$ for one of the $\bar{d}$ eigenvalues $\bar{\lambda}_1, \ldots, \bar{\lambda}_{\bar{d}}$.

Due to the linear independence of the exponential function (with distinct growth rates) it follows that $x \equiv \bar{x}$ is only possible, when there is at least one common eigenvalue (unless $x_0 = 0$). We can reorder the eigenvalues such that for some $r \geq 1$

$$\lambda_1 = \bar{\lambda}_1, \ldots, \lambda_r = \bar{\lambda}_r$$

and $\lambda_p \neq \bar{\lambda}_q$ for all $p, q > r$, then $x \equiv \bar{x}$ implies for $\ell = 1, 2, \ldots, r$

$$\sum_{j=1}^{\dim \mathcal{V}^1_{\lambda_\ell}} \sum_{i=1}^{k_{\ell,j}} \alpha_{\ell,j,i} \sum_{\eta=1}^{i} v_\eta^{\ell,j} \frac{t^{i-\eta}}{(i-\eta)!} = \sum_{j=1}^{\dim \bar{\mathcal{V}}^1_{\bar{\lambda}_\ell}} \sum_{i=1}^{\bar{k}_{\ell,j}} \bar{\alpha}_{\ell,j,i} \sum_{\eta=1}^{i} \bar{v}_\eta^{\ell,j} \frac{t^{i-\eta}}{(i-\eta)!} \qquad (8)$$

and, for all $\ell > r$ and all corresponding i, j

$$\alpha_{\ell,j,i} = 0, \quad \bar{\alpha}_{\ell,j,i} = 0.$$

By repeatedly taking time-derivatives of (8) and evaluating at $t = 0$ we obtain the following equalities for $\kappa = 0, \ldots, \kappa^\ell_{\max} := \max\{k_{\ell,j}, \bar{k}_{\ell,j}\} - 1$:

$$w^\ell_\kappa := \sum_{j=1}^{\dim \mathcal{V}^1_{\lambda_\ell}} \sum_{i=1}^{k_{\ell,j}} \alpha_{\ell,j,i} v^{\ell,j}_{i-\kappa} = \sum_{j=1}^{\dim \overline{\mathcal{V}}^1_{\bar{\lambda}_\ell}} \sum_{i=1}^{\bar{k}_{\ell,j}} \alpha_{\ell,j,i} \bar{v}^{\ell,j}_{i-\kappa};$$

here we use the convention that quantities indexed outside their actual domain are zero by definition. It then follows for all $\ell = 1, 2, \ldots, r$ and all $\kappa = 0, 1, 2, \ldots, \kappa^\ell_{\max}$:

$$\begin{aligned}(\mathcal{A} - \lambda \mathcal{E}_{\mathfrak{L}}) w^\ell_\kappa &= \sum_{j=1}^{\dim \mathcal{V}^1_{\lambda_\ell}} \sum_{i=1}^{k_{\ell,j}} \alpha_{\ell,j,i} (\mathcal{A} - \lambda \mathcal{E}_{\mathfrak{L}}) v^{\ell,j}_{i-\kappa} \\ &= \sum_{j=1}^{\dim \mathcal{V}^1_{\lambda_\ell}} \sum_{i=1}^{k_{\ell,j}} \alpha_{\ell,j,i} \mathcal{E} v^{\ell,j}_{i-\kappa-1} = \mathcal{E} w^\ell_{\kappa+1},\end{aligned}$$

where $w^\ell_{\kappa^\ell_{\max}+1} := 0$. An analogous calculation shows that

$$(\mathcal{A} - \lambda \mathcal{E}_{\overline{\mathfrak{L}}}) w^\ell_\kappa = \mathcal{E} w^\ell_{\kappa+1};$$

in particular, the tuple $(w^\ell_{\kappa^\ell_{\max}}, \ldots, w^\ell_1, w^\ell_0)$ (note the reversed order) satisfies the eigenvector chain condition (5) and we have shown that

$$x(0) = \sum_\ell^r w^\ell_0$$

is an element of $\operatorname{span} \mathfrak{C}_{\mathfrak{L}, \overline{\mathfrak{L}}}$.

□

Remark 5. *Note that existence of at least one common eigenvector is both necessary and sufficient for the existence of a nontrivial indiscernible initial condition (because any common eigenvector chain also contains a common eigenvector). But the set of initial conditions which are indiscernible are not limited to the span of common eigenvectors; they are spanned by common eigenvector chains. Only when all (common) eigenvalues are semi-simple (i.e. they do not correspond to Jordan blocks of size bigger than one), the span of common eigenvectors yields the whole space of indiscernible initial states.*

4. Indiscernible topological changes

In the design of a suitable network topology (with Laplacian matrix $\mathfrak{L}$) one goal could be to avoid the existence of any (nontrivial) indiscernible initial state with respect to many fault scenarios $\mathfrak{L} \to \overline{\mathfrak{L}}$. It is therefore meaningful to define the following properties of a topological change:

Definition 6. *For a coupled system* (4) *a topological change* $\mathfrak{L} \to \overline{\mathfrak{L}}$ *is called* always-discernible *if there is no (nontrivial) indiscernbible initial state and* possibly-indiscernible *if there exists a nontrivial indiscernible initial state.*

Note that we do not simply say that a topological change is discernible/indiscernible because the possibility to detect a topological change strongly depends on the initial state. Furthermore, even when a topological change is possibly-indiscernible, it will usually be discernible for *almost all* initial states, because the subspace of indiscernible initial states is a subspace of dimension usually smaller than n.

Our goal is now to provide a simple characterization of possible-indiscernibility which does not require the calculation of the whole set of indiscernible initial states. The following lemma is a key observation towards this goal:

Lemma 7. *Let* $(\lambda, v) \in \mathbb{C} \times \mathbb{C}^n \setminus \{0\}$ *be an eigenvalue-eigenvector pair of* $(\mathcal{E}, \mathcal{A}_{\mathfrak{L}})$. *Then* (λ, v) *is also an eigenvalue-eigenvector of* $(\mathcal{E}, \mathcal{A}_{\overline{\mathfrak{L}}})$ *if, and only if,*

$$v \in \ker(\mathcal{A}_{\overline{\mathfrak{L}}} - \mathcal{A}_{\mathfrak{L}})$$

Proof.

$$\begin{aligned}(\mathcal{A}_{\overline{\mathfrak{L}}} - \lambda \mathcal{E}) v = 0 \;&\Leftrightarrow\; (\mathcal{A}_{\overline{\mathfrak{L}}} - \mathcal{A}_{\mathfrak{L}} + \mathcal{A}_{\mathfrak{L}} - \lambda \mathcal{E}) v = 0 \\ &\overset{(\mathcal{A}_{\mathfrak{L}} - \lambda \mathcal{E}) v = 0}{\Leftrightarrow} (\mathcal{A}_{\overline{\mathfrak{L}}} - \mathcal{A}_{\mathfrak{L}}) v = 0.\end{aligned}$$

□

Utilizing the special structure of $\mathcal{A}_{\overline{\mathfrak{L}}} - \mathcal{A}_{\mathfrak{L}}$ we can derive the main result of this section:

Theorem 8. *Consider a family of DAEs of the form* (1) *connected by a network graph* $\mathfrak{G} = (\mathfrak{V}, \mathfrak{E})$ *with weighted Laplacian* $\mathfrak{L}$ *resulting in the overall system* (4), *which* *we assume to be regular. Any regularity-preserving removal/addition of the edge (i, j) is a possibly-indiscernible topological change if, and only if, there exists an eigenvector $v \in \mathbb{C}^n \setminus \{0\}$ of $(\mathcal{E}, \mathcal{A}_{\mathfrak{L}})$ with*

$$(\mathcal{C}v)_i - (\mathcal{C}v)_j \in \ker \begin{bmatrix} B_i \\ B_j \end{bmatrix}; \tag{9}$$

here $(\mathcal{C}v)_k \in \mathbb{R}^p$ (for k either i or j) denotes the k-th (block) entry of the vector $\mathcal{C}v \in \mathbb{R}^{Np}$ consisting in total of N entries of length p.

Proof. The addition/removal of edge (i, j) leads to a topological change $\mathfrak{L} \to \overline{\mathfrak{L}}$ with

$$\overline{\mathfrak{L}} = \mathfrak{L} \pm w_{ij}(e_i - e_j)(e_i - e_j)^\top;$$

hence $v \in \ker(\mathcal{A}_{\overline{\mathfrak{L}}} - \mathcal{A}_{\mathfrak{L}})$ if, and only if,

$$\mathcal{C}v \in \ker \mathcal{B}((e_i - e_j)(e_i - e_j)^\top \otimes I_p);$$

where we used bilinearity of the Kronecker product and $w_{ij} \neq 0$. It is easily seen that

$$\mathcal{B}((e_i - e_j)(e_i - e_j)^\top \otimes I_p) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \mathcal{B}_i & 0 & -\mathcal{B}_i & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\mathcal{B}_j & 0 & \mathcal{B}_j & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{10}$$

with suitably sized zero matrices. Together with Theorem 4 (in particular, Remark 5) and Lemma 7 this shows the claim of the Theorem. ☐

Remark 9. *The condition* (9) *derived in Theorem 8 will be satisfied if either $(\mathcal{C}v)_i = (\mathcal{C}v)_j$ or $(\mathcal{C}v)_i - (\mathcal{C}v)_j \in \ker \begin{bmatrix} B_i \\ B_j \end{bmatrix}$. If $(\mathcal{C}v)_i = (\mathcal{C}v)_j$ then there is no diffusion taking place at the edge (i, j) and as a result any addition or removal of edge between i-th and j-th vertex will go undetected. On the other hand, if $(\mathcal{C}v)_i - (\mathcal{C}v)_j \in \ker \begin{bmatrix} B_i \\ B_j \end{bmatrix}$ then the diffusive coupling between i-th and j-th vertex is unable to influence the dynamics at the respective vertices. Thus, any addition or removal of edge between i-th and j-th vertex will once again go undetected. Further, if we assume that input matrices B_i are of full column rank for all i, then condition* (9) *reduces to $(\mathcal{C}v)_i = (\mathcal{C}v)_j$.*

For illustrating above result, we recall from (Küsters et al., 2017, Ex. 6), an example of an electrical circuit which is a variant of the well known Wheatstone bridge.

Example 10. *Consider a circuit as shown in Figure 1.*

Here nodes 1 and 2 are connected to capacitors and hence lead to dynamic equations. On the other hand nodes 3 and 4 lead to algebraic equations. At each node, we have the following DAEs

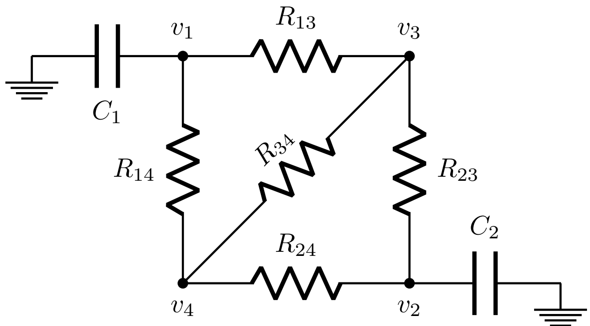

Figure 1: RC-circuit.

$$\begin{aligned} &\textit{Node 1}: & -C_1\dot{v}_1 &= u_1, & y_1 &= v_1, \\ &\textit{Node 2}: & -C_2\dot{v}_2 &= u_2, & y_2 &= v_2, \\ &\textit{Node 3}: & 0 &= u_3, & y_3 &= v_3, \\ &\textit{Node 4}: & 0 &= u_4, & y_4 &= v_4, \end{aligned}$$

along with the following conditions arising from the topology of the circuit

$$\begin{aligned} u_1 &= R_{14}(v_1 - v_4) + R_{13}(v_1 - v_3), \\ u_2 &= R_{24}(v_2 - v_4) + R_{23}(v_2 - v_3), \\ u_3 &= R_{13}(v_3 - v_1) + R_{23}(v_3 - v_2) + R_{34}(v_3 - v_4), \\ u_4 &= R_{14}(v_4 - v_1) + R_{24}(v_4 - v_2) + R_{34}(v_4 - v_3). \end{aligned}$$

The overall system equation is given by (4) *with*

$$\mathcal{E} = \begin{bmatrix} -C_1 & 0 & 0 & 0 \\ 0 & -C_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \ \mathcal{A} = 0_{4\times 4}, \ \mathcal{B} = I_4, \ \mathcal{C} = I_4$$

and

$$\mathfrak{L} = \begin{bmatrix} R_{13}+R_{14} & 0 & -R_{13} & -R_{14} \\ 0 & R_{23}+R_{24} & -R_{23} & -R_{24} \\ -R_{13} & -R_{23} & R_{13}+R_{23}+R_{34} & -R_{34} \\ -R_{14} & -R_{24} & -R_{34} & R_{14}+R_{24}+R_{34} \end{bmatrix}.$$

In this case, equation (4) *reduces to*

$$\mathcal{E}\dot{v}(t) = \mathfrak{L}v(t). \tag{11}$$

Assuming equal values of magnitude one for all the resistances and capacitances in this circuit, we compute the eigenvalues and eigenvectors of the matrix pair $(\mathcal{E}, \mathfrak{L})$. There are two finite eigenvalues $\lambda_1 = 0$ and $\lambda_2 = -2$ with corresponding eigenvectors

$$v^1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad v^2 = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}.$$

Clearly, both the eigenvectors are such that $(\mathcal{C}v)_3 - (\mathcal{C}v)_4 = 0$ is satisfied. Thus, by Theorem 8, the addition/removal of edge $(3, 4)$ is undetectable for "any" consistent initial value.

5. Indiscernibility for homogeneous networks

For homogenous networks, it is possible to simplify the result further. In this case, we have identical differential equations connected by a graph $\mathfrak{G} = (\mathfrak{V}, \mathfrak{E})$ with weighted Laplacian $\mathfrak{L}$. Substituting $E_i = E$, $A_i = A$, $B_i = B$, $C_i = C$ and $n_i = \overline{n}$ for all $i \in \mathfrak{V}$ in (4), we are able to write the overall dynamics in a simplified form as follows.

$$\mathcal{E}\dot{x} = \mathcal{A}_{\mathfrak{L}} x, \tag{12}$$

where

$$\mathcal{E} := (I_N \otimes E),$$
$$\mathcal{A}_{\mathfrak{L}} := (I_N \otimes A) - \mathfrak{L} \otimes BC.$$

As a result, indiscernibility of homogenous DAE network can be partly described in terms of the eigenvectors of the Laplacian of the connection graph under certain observability assumptions. For that we first note the following properties of eigenvalue-eigenvectors pairs of $(\mathcal{E}, \mathcal{A}_{\mathfrak{L}})$ in the homogeneous case.

Lemma 11. *Let $\alpha_1, \alpha_2, \ldots, \alpha_N \in \mathbb{R}$ be the N real eigenvalues (counting multiples) of the symmetric Laplacian $\mathfrak{L}$. Then, for $(\mathcal{E}, \mathcal{A}_{\mathfrak{L}})$ as above,*

$$\operatorname{spec}(\mathcal{E}, \mathcal{A}_{\mathfrak{L}}) = \bigcup_{i=1}^{N} \operatorname{spec}(E, A - \alpha_i BC). \tag{13}$$

Further, let v be a (generalized) eigenvector of $(\mathcal{E}, \mathcal{A}_{\mathfrak{L}})$ for an eigenvalue λ of $(\mathcal{E}, \mathcal{A}_{\mathfrak{L}})$ with $\lambda \in \operatorname{spec}(E, A - \alpha BC)$ for some eigenvalue α of $\mathfrak{L}$. Then

$$v = z \otimes w \tag{14}$$

where z is an eigenvector of $\mathfrak{L}$ corresponding to α and w is a (generalized) eigenvector of $(E, A - \alpha BC)$ corresponding to λ. Moreover, if v and w are generalized eigenvectors, then they have the same rank.

Proof. Since $\mathfrak{L}$ is symmetric there exists an orthogonal coordinate transformation S such that $S^\top \mathfrak{L} S = \Lambda = \operatorname{diag}\{\alpha_1, \ldots, \alpha_N\}$. Choose a coordinate transformation $D := S \otimes I_{\overline{n}}$ for $\mathcal{A}_{\mathfrak{L}}$. From the properties of the Kronecker product $(X \otimes Y)^\top = X^\top \otimes Y^\top$ and $(X \otimes Y)(Z \otimes W) = (XZ \otimes YW))$ it follows that

$$\begin{aligned} D^\top (I_N \otimes A) D &= (S^\top \otimes I_{\overline{n}})(I_N \otimes A)(S \otimes I_{\overline{n}}) \\ &= (S^\top \otimes I_{\overline{n}})(S \otimes A) = I_N \otimes A, \end{aligned}$$

$$D^\top \mathcal{E} D = \mathcal{E}$$

and

$$\begin{aligned} D^\top (\mathfrak{L} \otimes BC) D &= (S^\top \otimes I_{\overline{n}})(\mathfrak{L} \otimes BC)(S \otimes I_{\overline{n}}) \\ &= \Lambda \otimes BC. \end{aligned}$$

Therefore,

$$\begin{aligned} D^\top \mathcal{A}_{\mathfrak{L}} D &= D^\top (I_N \otimes A) D - D^\top (\mathfrak{L} \otimes BC) D \\ &= (I_N \otimes A) - (\Lambda \otimes BC) \\ &= \operatorname{diag}\{A - \alpha_1 BC, A - \alpha_2 BC, \ldots, A - \alpha_N BC\}, \end{aligned}$$

which shows (13).

Note that for any eigenvalue-eigenvector pair $(\alpha, z) \in \mathbb{R} \times \mathbb{R}^N$ of $\mathfrak{L}$ and any $(\lambda, w) \in \mathbb{C} \times \mathbb{C}^{\overline{n}}$ we have

$$\begin{aligned} &(\mathcal{A}_{\mathfrak{L}} - \lambda \mathcal{E})(z \otimes w) \\ &\quad = \big((I_N \otimes A) - (\mathfrak{L} \otimes BC)\big)(z \otimes w) - \lambda(z \otimes Ew) \\ &\quad = (z \otimes Aw) - (\mathfrak{L}z \otimes BCw) - \lambda(z \otimes Ew) \\ &\quad = (z \otimes Aw) - (\alpha z \otimes BCw) - \lambda(z \otimes Ew) \\ &\quad = z \otimes \big((A - \alpha BC - \lambda E)w\big) = 0 \end{aligned} \tag{15}$$

and, generalizing,

$$(\mathcal{A}_{\mathfrak{L}} - \lambda \mathcal{E})^i (z \otimes w) = z \otimes \big((A - \alpha BC - \lambda E)^i w\big).$$

This shows that any (generalized) eigenvalue-eigenvector pairs (α, z) and (λ, w) of $\mathfrak{L}$ and $(E, A - \alpha BC)$, respectively, leads to a generalized eigenvector $v = z \otimes w$ of $(\mathcal{E}, \mathcal{A}_{\mathfrak{L}})$ of the same rank. Furthermore, the dimension of

$$\operatorname{span}\left\{ z \otimes w \;\middle|\; \begin{array}{l} \exists \alpha \in \operatorname{spec} \mathfrak{L} : (\mathfrak{L} - \alpha I) z = 0 \text{ and} \\ \exists \lambda \in \operatorname{spec}(E, A - \alpha BC)\ \exists i \in \mathbb{N} : \\ (A - \alpha BC - \lambda E)^i w = 0 \end{array} \right\}$$

is $n = N \cdot \overline{n}$, hence there is a bijection between the set of all eigenvectors of $(\mathcal{E}, \mathcal{A}_{\mathfrak{L}})$ and the set of eigenvectors given by (14), which concludes the proof. □

Using Lemma 11 we now get the following result for indiscernible initial conditions of a homogenous DAE network.

Theorem 12. *Consider a family of identical DAEs* (1) *of the form*

$$\begin{aligned} E\dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}$$

connected via the diffusive coupling (2) *by a network with weighted Laplacian $\mathfrak{L}$ resulting in the overall system* (12)*, which we assume to be regular. Suppose furthermore that $B \neq 0$, $C \neq 0$, $BC \neq 0$ and that (E, A, C) is observable in the behavioral sense,* i.e, $\operatorname{rank}\left[\begin{smallmatrix} \lambda E - A \\ C \end{smallmatrix}\right] = n$ *for all $\lambda \in \mathbb{C}$, see e.g. Berger et al. (2017). Then, any regularity-preserving removal/addition of the edge (i, j) is a possibly-indiscernible topological change if, and only if, either there exists an eigenvector $z \in \mathbb{C}^N \setminus \{0\}$ of $\mathfrak{L}$ such that*

$$z_i = z_j \tag{16a}$$

or there exists an eigenvector $w \in \mathbb{C}^{\overline{n}} \setminus \{0\}$ of (E, A) with

$$Cw \in \ker B. \tag{16b}$$

In both cases any indiscernible initial state takes the form $v = z \otimes w$ as in (14).

Proof. From Theorem 4 (see also Remark 5), the existence of indiscernible states is equivalent to the existence of an eigenvalue-eigenvector pair (λ, v) common to $(\mathcal{E}, \mathcal{A}_{\mathfrak{L}})$ and $(\mathcal{E}, \mathcal{A}_{\overline{\mathfrak{L}}})$. By invoking Lemma 7, we know that existence of a common eigenvector v of $(\mathcal{E}, \mathcal{A}_{\mathfrak{L}})$ and $(\mathcal{E}, \mathcal{A}_{\overline{\mathfrak{L}}})$ implies $(\mathcal{A}_{\mathfrak{L}} - \mathcal{A}_{\overline{\mathfrak{L}}})v = 0$. By Lemma 11 the latter can be rewritten as

$$(\mathfrak{L} - \overline{\mathfrak{L}})z \otimes BCw = 0,$$

where z and w satisfy, for some $\alpha \in \mathbb{C}$,

$$v = z \otimes w, \tag{17a}$$
$$(A - \alpha BC)w = \lambda E w, \tag{17b}$$
$$\mathfrak{L}z = \alpha z. \tag{17c}$$

Hence we either have $BCw = 0$ or $(\mathfrak{L} - \overline{\mathfrak{L}})z = 0$. If $BCw = 0$ then by (17b) we have that w is an eigenvector of (E, A). Due to the assumption that $BC \neq 0$ and by observability of (E, A, C), we have $Cw \neq 0$. Thus, $Cw \in \ker B$.

If instead $(\mathfrak{L} - \overline{\mathfrak{L}})z = 0$ then $(e_i - e_j)(e_i - e_j)^\top z = 0$, and thus we get $z_i = z_j$.

To show the converse claim, assume that either (16a) or (16b) holds. Then $(\mathcal{A}_{\mathfrak{L}} - \mathcal{A}_{\overline{\mathfrak{L}}})v = 0$ for any v of the form $v = z \otimes w$ and in view of Lemma 7 together with Remark 5 it suffices to show that in both cases we can construct $v = z \otimes w$ in such a way that v is an eigenvector of $(\mathcal{E}, \mathcal{A}_{\mathfrak{L}})$. In the first case, let $\alpha \in \mathbb{C}$ be the corresponding eigenvalue for the eigenvector z of $\mathfrak{L}$ and choose any eigenvector w of $(E, A - \alpha BC)$. Then (15) shows that $v = z \otimes w$ is an eigenvector of $(\mathcal{E}, \mathcal{A}_{\mathfrak{L}})$. In the second case we have $BCw = 0$, hence w is also an eigenvector of $(E, A - \alpha BC)$ for any α. Hence we may choose any eigenvector z of $\mathfrak{L}$ and (15) again shows that $v = z \otimes w$ is an eigenvector of $(\mathcal{E}, \mathcal{A}_{\mathfrak{L}})$. □

We note following aspects of the conditions obtained in Theorem 12.

1. First note that the two conditions in (16) have a very distinct feature which is as follows. The condition $z_i = z_j$ only depends on the Laplacian of network graph. On the other hand existence of eigenvector w of the pair (E, A) for which $Cw \in \ker B$ is solely a property of the individual subsystems. Thus, Theorem 12 offers two independent indiscernibility conditions – one on the network and the other one on each subsystem.

2. The fact, that existence of an eigenvector w of the pair (E, A) for which $Cw \in \ker B$ leads to indiscernibility, is quite intuitive because of the following reason. If we set the initial conditions of the i-th and j-th subsystem to be w, then the diffusive coupling between i-th and j-th node will be annihilated by the matrix B. Thus it will not have any effect on the overall dynamics and hence addition/removal of edge (i, j) will be unnoticed.

3. If we assume that the matrix B is full column rank then the condition for indiscernibility (16) depends only on the Laplacian eigenvector z which must satisfy $z_i = z_j$. The Laplacian matrix $\mathfrak{L}$ always has at least one eigenvalue which is zero with corresponding eigenvector $z = (1, 1, \ldots, 1)^\top$. The condition $z_i = z_j$ is always satisfied for this eigenvector. As a consequence, any topological change of a homogeneous network is necessarily possibly-indiscernible. This special eigenvector corresponds to the situation where all subsystems start with the same initial value; as a consequence, the diffusive coupling is zero and a topological variation has no effect on the dynamics.

4. In order to have diffusive coupling, it is necessary to assume $B \neq 0$, $C \neq 0$ and $BC \neq 0$. Moreover, the observability assumption on (E, A, C) is also required, as otherwise it is possible to set the initial conditions of each subsystem in its unobservable subspace. Consequently, the ensuing dynamics will have no effect on the output of the individual subsystems and, therefore, will have no effect on the overall network behavior.

6. Regularity preserving topological changes

Our main results (Theorems 4 and 8) assume regularity preserving topological changes. Without the regularity assumption, uniqueness of solutions does not hold any more, so that even Definition 1 becomes meaningless. In general, it is not a trivial task to decide whether the overall DAE (4) is regular or not. The following examples show that it is possible that although all subsystems are regular, the coupled system loses regularity; and, on the other hand, the coupled system can be regular although the individual subsystems are not regular.

Example 13 (Loss of regularity by coupling[2]). *Consider two DAE systems given by*

$$\begin{aligned} 0 &= x_1 + u_1, \qquad & 0 &= x_2 + u_2, \\ y_2 &= x_1, & y_2 &= x_2, \end{aligned}$$

which are clearly regular. However, under diffusive coupling with coupling strength $w_{12} = w_{21} = \frac{1}{2}$, *the overall system reads as*

$$\begin{aligned} 0 &= \tfrac{1}{2}x_1 + \tfrac{1}{2}x_2, \\ 0 &= \tfrac{1}{2}x_1 + \tfrac{1}{2}x_2, \end{aligned}$$

which is not regular.

Example 14 (Regularization by coupling). *Consider the following three DAE systems*

$$\begin{aligned} 0 &= x_1, \quad & 0 &= x_2, \quad & 0 &= u_3 \\ y_1 &= x_1, & y_2 &= x_2, & y_3 &= x_3 \end{aligned}$$

where the third DAE is not regular (because $E_3 = 0 = A_3$*). However, under diffusive coupling with* $w_{12} = w_{21} = 0$, $w_{13} = w_{31} = R_1 > 0$, *and* $w_{23} = w_{32} = R_2 > 0$, *the overall DAE reads as*

$$\begin{aligned} 0 &= x_1 \\ 0 &= x_2 \\ 0 &= (R_1 + R_2)x_3 - R_1 x_1 - R_2 x_2 \end{aligned}$$

[2]We thank Ferdinand Küsters for providing this nice example.

which is regular, for any positive choices of R_1 and R_2. Another example is the Wheatstone bridge as in Example 10 above.

Under the (reasonable) assumption that the nominal coupled DAE $(\mathcal{E}, \mathcal{A}_{\mathfrak{L}})$ is regular, one can interpret any topological change $\mathfrak{L} \to \overline{\mathfrak{L}}$ as an introduction of an additional feedback term:

$$\mathcal{A}_{\overline{\mathfrak{L}}} = \mathcal{A}_{\mathfrak{L}} \pm \mathcal{B}\underbrace{((\overline{\mathfrak{L}} - \mathfrak{L}) \otimes I_p)\mathcal{C}}_{:=\mathcal{F}}.$$

Therefore, we can use the following sufficient condition for regularity:

Lemma 15 ((Bunse-Gerstner et al., 1992, Thm. 11)). *Consider a regular matrix pair $(E, A) \in \mathbb{R}^{n\times n} \times \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times p}$. If*

$$\operatorname{rank} E = \operatorname{rank}[E, B],$$

then $(E, A + BF)$ is regular for all feedback matrices $F \in \mathbb{R}^{p\times n}$.

Therefore, we arrive immediately at the following sufficient condition for regularity preservation:

Corollary 16. *Consider a regular coupled DAE (4). If $\operatorname{rank} \mathcal{E} = \operatorname{rank}[\mathcal{E}, \mathcal{B}]$ then any topological change $\mathfrak{L} \to \overline{\mathfrak{L}}$ is regularity preserving.*

Due to the block structure of $\mathcal{E}$ and $\mathcal{B}$, the condition $\operatorname{rank} \mathcal{E} = \operatorname{rank}[\mathcal{E}, \mathcal{B}]$ is equivalent to $\operatorname{rank} E_i = \operatorname{rank}[E_i, B_i]$ for all $i = 1, 2, \ldots, N$. In fact, by considering a removal/addition/change of a single edge (i, j) the regularity-preservation condition reduces in view of (10) to the two sufficient conditions

$$\operatorname{rank} E_i = \operatorname{rank}[E_i, B_i] \quad \text{and} \quad \operatorname{rank} E_j = \operatorname{rank}[E_j, B_j].$$

In other words, any topological change involving only edges between nodes which satisfy the rank condition $\operatorname{rank} E_i = \operatorname{rank}[E_i, B_i]$ preserves regularity of the corresponding DAE.

7. Conclusions

Understanding when a topological variation cannot be detected is fundamental for monitoring, and eventually controlling, complex networks. In this paper, we have studied this problem for a class of linear DAE networks, using tools from control theory. The results, which account for multivariable and heterogenous dynamics, show that the problem can be fully characterized in terms of generalized eigenspaces. Moreover, under rather mild conditions, the existence of indiscernible topological changes can be assessed by only looking at the properties of the nominal network configuration.

Our results represent only a first step towards the development of algorithms for detecting and isolating network topological changes. Yet, the results provide many quantitive insights into the problem. For example, they indicate that under homogeneity and obervability assumptions, the dynamics at the nodes do not play any role in determining the set of indiscernible states. In this case, assessing discernibility is not more difficult than for a simple integrator network Battistelli and Tesi (2015).

We envision two main directions for future research, both of major practical value. First, understanding how the present results can be extended to the case where only a subset of nodes is available for measurements. Second, extending the analysis so as to incorporate a notion of "degree" of discernibility, as done in Baglietto et al. (2014). In fact, it is natural to expect that states close (in terms of Euclidean distance) to the indiscernibility set are in practice as much critical as indiscernible states. A notion of "degree" of discernibility would then help us to identify regions of the state space where detecting topological changes is more easy or difficult to obtain.

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