Analogue Implementation of the Funnel Controller

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In many tracking control problems, pre-specified bounds for the evolution of the tracking error should be met. The 'funnel controller' addresses this requirement and guarantees transient performance for a fairly large class of systems. In addition, only structural assumptions on the underlying system are made; the exact knowledge of the system parameters is not required. This is in contrast to most classical controllers where only asymptotic behaviour can be guaranteed and the system parameters must be known or estimated. Until now, the funnel controller was only studied theoretically. We will present the results of an analogue implementation of the funnel controller. The results show that the funnel controller works well in reality, i.e. it guarantees the pre-specified error bounds. The implementation is an analogue circuit composed of standard devices and is therefore suitable for a broad range of applications.

1 Introduction

The *funnel controller* was developed to solve tracking problems for a class of nonlinear systems described by functional differential equations such that prespecified error bounds are met.

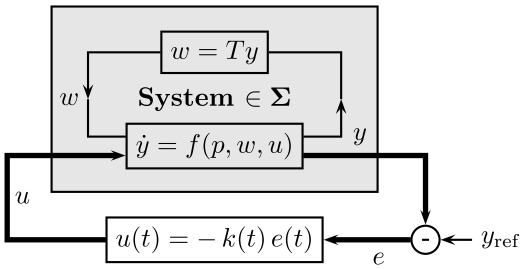

Fig. 1 Universal error feedback control.

The system class Σ for which the funnel controller is shown to work is described in detail in [3]. The main property of systems belonging to Σ is that they are high-gain stabilizable, i.e. large inputs u yields large derivatives of y in the "right" direction. The system class Σ allows for bounded disturbance p, and functional operators T such as hysterises and delays. Linear minimum-phase systems with relative degree one belong to Σ as well as a broad class of infinite dimensional linear systems [3]. It should be highlighted here, that the funnel controller does not depend on the specific system, but is the same for all systems in class Σ; the same control-law will achieve the prespecified error bounds for a simple finite dimensional linear system, a highly nonlinear system or a infinite dimensional linear system.

The desired error bounds are given by a region $\mathcal{F} \subset \mathbb{R}_{\geq 0} \times \mathbb{R}^n$, the funnel, where $n \in \mathbb{N}$ is the dimension of the output y of the system (see Figure 2). The funnel controller ensures that the error $e = y - y_{\text{ref}}$ between the system's output and the reference signal y_{ref} fulfills $(t, e(t)) \in \mathcal{F}$ for all $t \geq 0$.

The funnel $\mathcal{F}$ plays two rôles: it governs the asymptotic behaviour and it describes precisely the desired transient behaviour of the error. Besides some smoothness conditions (see [3] for details) there are no restrictions on the shape of the funnel, it is, for example, possible to allow for larger error bounds at specific times in view of a priori known disturbances such as a regular re-calibration of sensors.

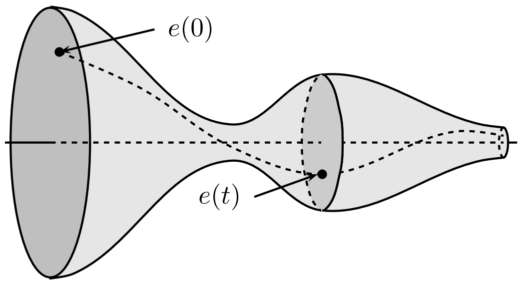

Fig. 2 Performance funnel $\mathcal{F}$.

For a prespecified funnel $\mathcal{F}$ the funnel controller is given by the simple proportional error feedback

$$u(t) = -k(t)\,e(t), \quad t \geq 0,$$

where the time varying gain $k(\cdot)$ is defined as

$$\boxed{k(t) = K_{\mathcal{F}}\big(t, e(t)\big).}$$

The continuous nonlinear and memoryless function $K_{\mathcal{F}} : \mathcal{F} \to \mathbb{R}$ is the "heart" of the funnel controller. If it exhibits two specific properties, it is shown that the tracking control is solved, i.e. the error evolves within the funnel for the whole time [3, Thm. 2]. The first of which ensures that, if $(t, e(t))$ approaches the funnel boundary, then the gain attains values sufficiently large to preclude boundary contact, and the second of which obviates the need for large gain values away from the funnel boundary.

The funnel controller was first introduced in [2], where $K_{\mathcal{F}}$ had a very special form. A generalization of this approach was studied in [3]. A first step in the direction of applications was done in [4], where the funnel controller was applied to a model of chemical reactors in the presence of input saturation. Nevertheless only simulations were carried out. Until recently no "real world implementation" of the funnel controller was done.

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2 Implementation of the funnel controller with analogue circuits

We restrict ourselves to the scalar case, i.e. the output y of the system is a real-valued function. The funnel is given by

$$\mathcal{F} := \{\ (t,e)\ \mid t \geq 0, \varphi(t)|e| < 1\ \},$$

where $\varphi : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is any bounded differentiable function with bounded derivative and $\varphi(t) > 0$ for all $t > 0$. As gain function $K_{\mathcal{F}}$ we consider, for $(t,e) \in \mathcal{F}$,

$$K_{\mathcal{F}}(t,e) = \frac{\varphi(t)}{1-\varphi(t)|e|} = \frac{1}{1/\varphi(t) - |e|},$$

i.e. $K_{\mathcal{F}}$ is the reciprocal of the "vertical" distance of the error to the funnel boundary.
The aim is to design a circuit which implements the control law

$$u(t) = -\frac{\varphi(t)e(t)}{1-\varphi(t)\big|e(t)\big|}.$$

The above control law can be represented as a block diagram, where the output of a blocks stands for point-wise multiplication of the input with the value of the block.

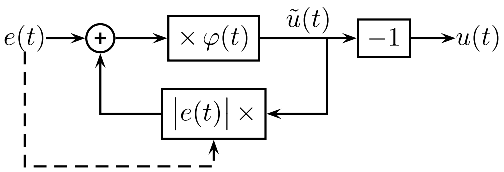

Fig. 3 Control law as block diagram.

Note that $\varphi(t)$ is an external signal which can be chosen arbitrarily by the user. Therefore the control law can be realized by the following circuit:

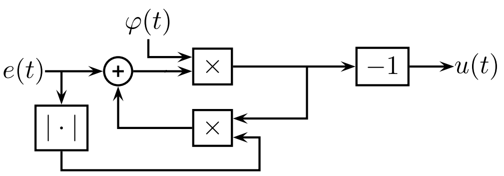

Fig. 4 Structure of the circuit implementation.

3 Experiments

For the implementation, we used standard circuits for the negation, the adder and the full wave rectifier (producing the absolute value) as can be found in [1]. In particular, we used the multiplier `MPY634` and the operational amplifiers `µA741` and `LM324A`, each with a supply voltage of 15V. As reference system we considered a circuit which has the ideal transfer-function

$$G(s) = \frac{c_1}{s+c_2}, \quad s \in \mathbb{C},$$

where c_1, c_2 are positive. The following figures show the results of the experiment. In all figures the time axis is in seconds and the values are given in volts. For the funnel boundary we chose an increasing function $\varphi(\cdot)$ which settles at a value of approximately 2.3V.

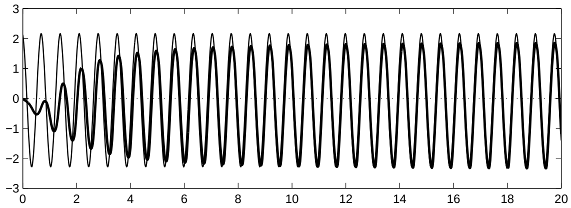

Fig. 5 Controlled output (thick line) and reference signal.

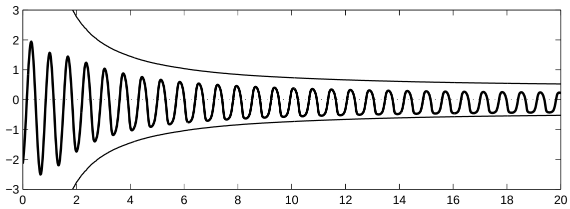

Fig. 6 Error (thick line) and funnel boundary $1/\varphi(t)$.

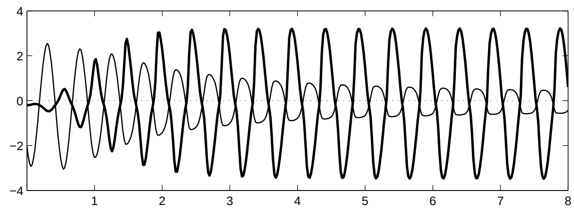

Fig. 7 Control input (thick line) and error.

As can be seen clearly, the funnel achieves tracking of the reference signal with the prespecified accuracy. Due to the circuit design it was not possible to measure the value of the gain $k(t)$, nevertheless the gain function can be calculated by $k(t) = -u(t)/e(t)$. Note that the gain was set to zero for small errors to suppress noise induced peaks. The obtained gain function is shown in the following figure:

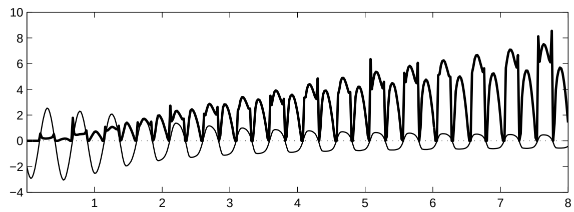

Fig. 8 Gain function (thick line) and error.

Note that the final maximal value of the gain function settles at a value of 18.

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