

# Observer Design for Detectable Switched DAE

Stephan Trenn

joint work with Aneel Tanwani (LAAS-CNRS, Toulouse)

Technomathematics group, University of Kaiserslautern, Germany  
supported by DFG-Grant TR 1223/2-1

IFAC 2017 World Congress, Toulouse, France  
10 July 2017, MoP11.2, 16:20–16:40





# An example

$$\dot{x}_1 = 0$$

$$\dot{x}_2 = 0$$

$$\dot{x}_3 = 0$$

$$y = x_1 + x_2$$

$$\dot{x}_1 = -x_2$$

$$\dot{x}_2 = x_1$$

$$0 = x_3 - x_2$$

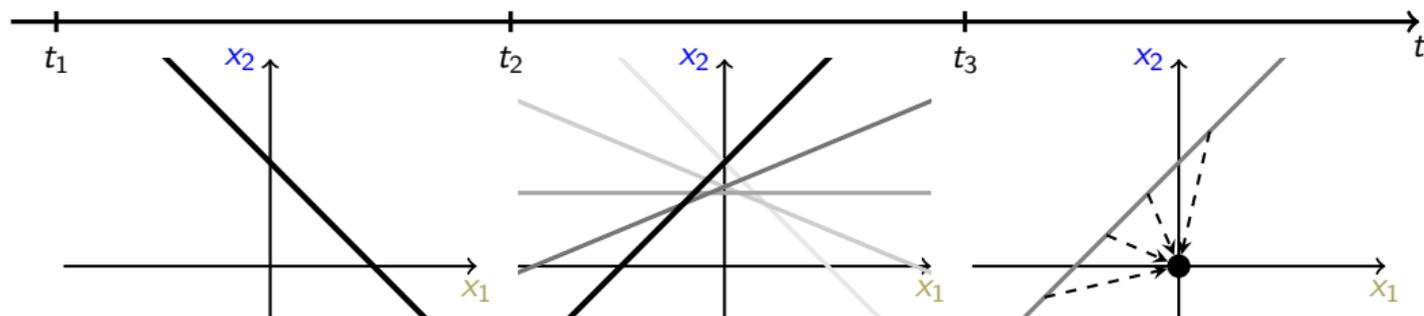
$$y = 0$$

$$\dot{x}_1 = -x_2$$

$$0 = x_1$$

$$\dot{x}_3 = 0$$

$$y = x_2$$



Can we reconstruct the state (at the final time) from the output?

$$y(t_2^-) = x_1(t_2^-) + x_2(t_2^-)$$

$$= x_1(t_2^+) + x_2(t_2^+) \checkmark$$

$$x_3(t_2^-) = ?$$

$$t_3 - t_2 = \pi/4$$

$$x_2(t_3^-) = x_1(t_2^+) ? \checkmark$$

$$x_1(t_3^-) = -x_2(t_2^+) \checkmark$$

$$y[t_3] = x_2[t_3]$$

$$= x_1(t_3^-) \delta_{t_3} \checkmark$$

$$x_3(t_3^-) = x_2(t_3^-) ? \checkmark$$

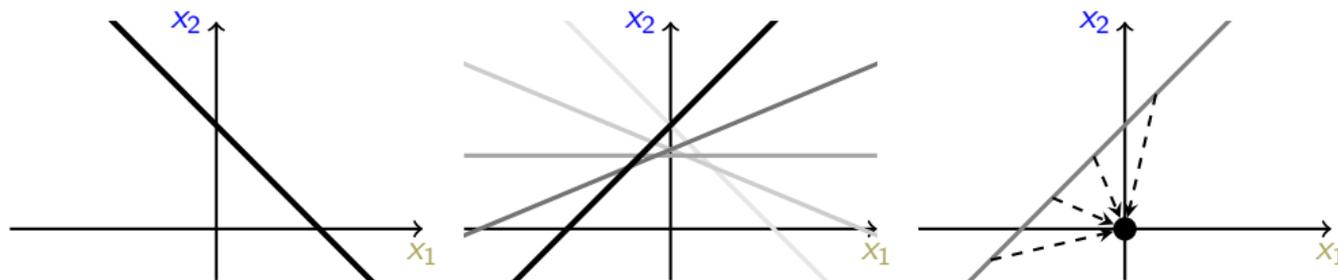
$$x_1(t_3^+) = 0 \checkmark$$

$$x_2(t_3^+) = 0 \checkmark$$

$$x_3(t_3^+) = x_3(t_3^-) ? \checkmark$$

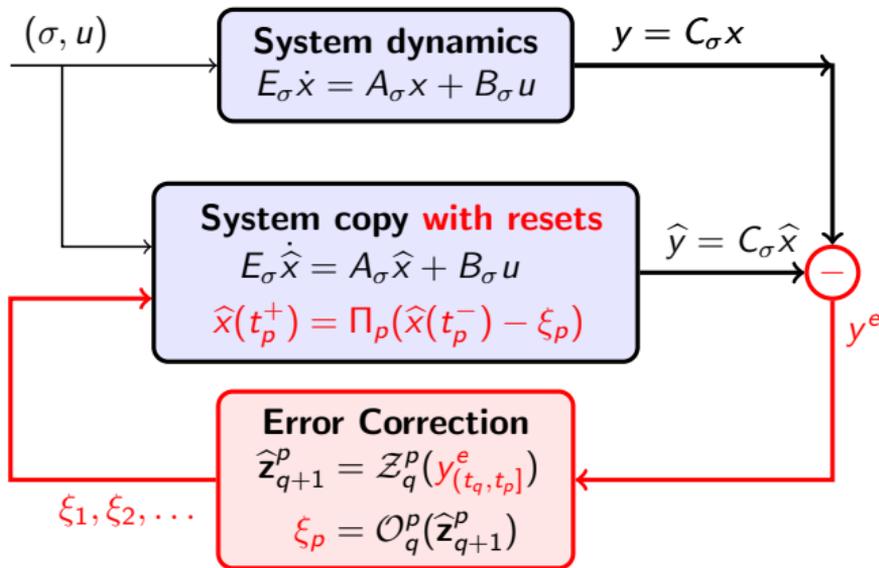


## Discussion of example



- **Non-observability** of individual modes
- **Several switches** are necessary for reconstruction of states
- $y$  on  $[t_1, t_3]$  does not determine  $x(t_1^\pm)$  but  $x(t_3^+)$   
 ⇒ **Observability vs. Determinability**
- **Partial knowledge** of states have to be propagated in time and adequately combined with each other
- **Algebraic constraints** of states need to be utilized
- **Dirac impulses** contribute to reconstruction of state

# Observer structure from [Tanwani & T., Automatica 2016]



**When does this work?**

Persistent Determinability

**How does it work?**

Choice of  $\xi_p$



# Determinability vs. Detectability

## Known for classical observer design:

Determinability (i.e. observability) sufficient but **not necessary** for existence of observer!

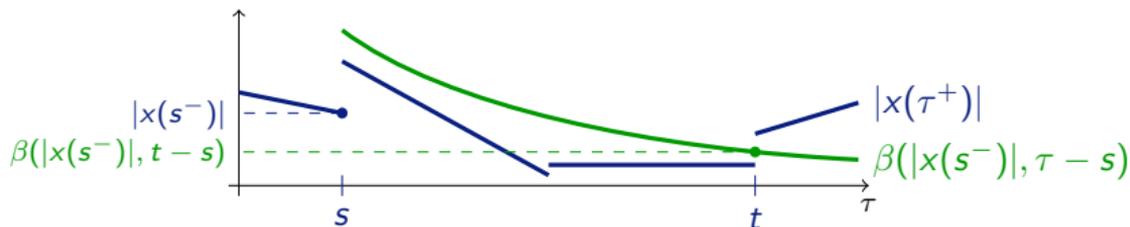
→ **detectability** necessary and sufficient

## Definition ( $[s, t)$ -Detectability, cf. [Tanwani & T., CDC'15])

(swDAE) is called  $[s, t)$ -detectable  $:\Leftrightarrow \forall (x, u \equiv 0, y \equiv 0)$  solutions on  $[t, s)$ :

$\exists$   $\mathcal{KL}$ -function  $\beta : \mathbb{R}^n \times [0, \infty) \rightarrow \mathbb{R}^n$  with  $\beta(r, t-s) < r$  and

$$|x(\tau^+)| \leq \beta(|x(s^-)|, \tau - s) \quad \forall \tau \in [s, t)$$





# Uniform interval-detectability

## Theorem (Interval-detectability $\Rightarrow$ global detectability)

Consider (swDAE) with switching times  $t_0 < t_1 < \dots$  and assume  $\exists p_0 < p_1 < \dots$  such that

- (swDAE) is  $[t_{p_i}, t_{p_{i+1}})$ -detectable with  $\mathcal{KL}$ -function  $\beta_i \quad \forall i \in \mathbb{N}$
- $\exists$  uniform  $\alpha \in (0, 1)$ :  $\beta_i(r, t_{p_{i+1}} - t_{p_i}) \leq \alpha r \quad \forall r > 0 \quad \forall i \in \mathbb{N}$
- $\exists$  uniform  $M > 0$ :  $\beta_i(r, 0) < Mr \quad \forall r > 0 \quad \forall i \in \mathbb{N}$

$\Rightarrow \quad \forall (x, u, y), (\bar{x}, u, y)$  solutions of (swDAE):

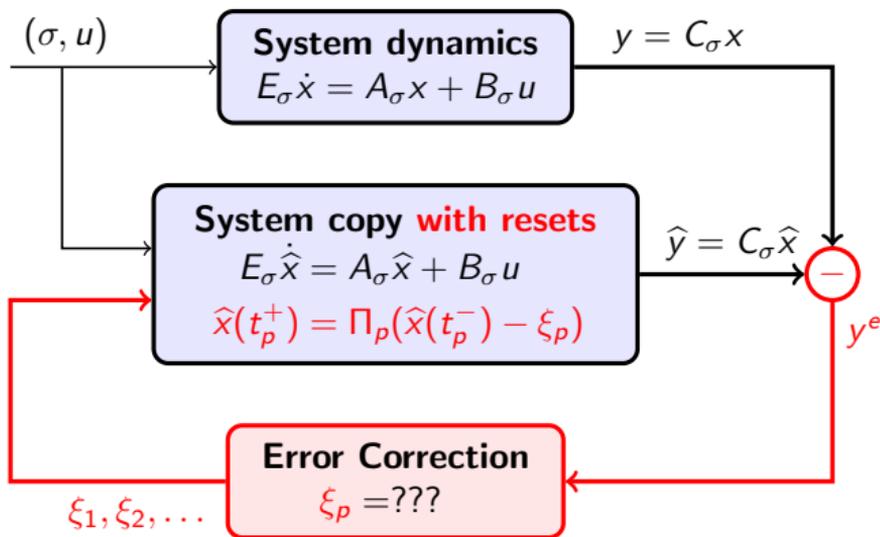
$$x(t^\pm) - \bar{x}(t^\pm) \rightarrow 0 \quad \text{as } t \rightarrow \infty$$

## Remarks

- No upper bounds on length of detectable intervals
- Implicit dwell time condition via interval-detectability condition
- Result seems to be new even for switched ODEs



# Observer structure



**When does this work?**

Uniform interval detectability

**How does it work?**

Choice of  $\xi_p$



# Novel Estimation Correction Necessary

$$\dot{x}_1 = 0$$

$$\dot{x}_2 = 0$$

$$\dot{x}_3 = 0$$

$$\mathbf{y} = x_1$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = 0$$

$$\dot{x}_3 = -x_3 + x_2$$

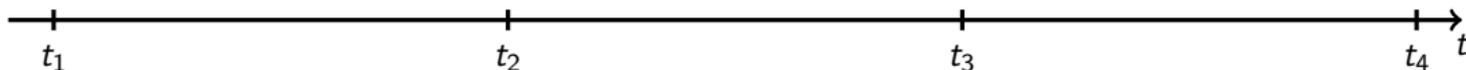
$$\mathbf{y} = 0$$

$$\dot{x}_1 = 0$$

$$\dot{x}_2 = 0$$

$$\dot{x}_3 = 0$$

$$\mathbf{y} = x_1$$



- System is detectable on  $[t_1, t_4)$
- $(x_1, x_2)$  is observable on  $[t_1, t_4)$ ,  $x_3$  is not observable
- **Simple idea:** Use known observer to update  $(x_1, x_2)$  at end of interval, keep  $x_3$  unchanged

**Simple approach not feasible**

Estimation error for  $x_3$  arbitrarily large!



# Novel Estimation Correction Procedure

For each detectability interval do:

**Step 1:** Collect local observability data **synchronously** to systems dynamics

- Basically the same approach as in our previous observer design
- Main difference: local estimation needed at beginning of each subinterval

**Step 2:** Propagate back collected information for **estimation correction at beginning of interval**

- Based on recursively defined **unobservability spaces**,  $k = q, q - 1, \dots, p$ ,

$$\mathcal{N}_k^q := \{ e(t_k^-) \mid e \text{ solves error dynamics on } [t_k, t_q] \}$$

- Results in a “virtual” error correction  $\xi^{\text{left}}$  at the beginning of the interval

**Step 3:** Propagate  $\xi^{\text{left}}$  **forward** to obtain estimation correction at end of interval

- $\xi = e(t_q^-)$  where  $e$  solves error dynamics  $E_\sigma \dot{e} = A_\sigma e$ ,  $e(t_p^-) = \xi^{\text{left}}$
- No input and output storage needed