A preliminary result on synchronization of heterogeneous agents via funnel control

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54th IEEE Conference on Decision and Control, CDC 2015, Osaka, Japan
Wednesday, 16th December 2015, WeA07.6, 11:40

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Problem statement

Given

- N agents with individual scalar dynamics:

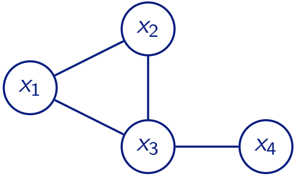
$$\dot{x}_i = f_i(t, x_i) + u_i$$

- undirected connected coupling-graph $G = (V, E)$
- agents know average of neighbor states

Desired

Control design for practical synchronization

$$x_1 \approx x_2 \approx \ldots \approx x_n$$



$$\bar{x}_1 := \frac{1}{2}(x_2 + x_3)$$

$$\bar{x}_2 := \frac{1}{2}(x_1 + x_3)$$

$$\bar{x}_3 := \frac{1}{3}(x_1 + x_2 + x_4)$$

$$\bar{x}_4 := x_3$$

A „high-gain" result

Let $\mathcal{N}_i := \{ j \in V \mid (j,i) \in E \}$ and $d_i := |\mathcal{N}_i|$.

Diffusive coupling

$$u_i = -k \sum_{j \in \mathcal{N}_i} (x_i - x_j) = -k d_i (x_i - \overline{x}_i)$$

Theorem (Practical synchronization, Kim et al. 2013)

Assumptions: G connected, all solutions of average dynamics

$$\dot{s}(t) = \frac{1}{N} \sum_{i=1}^{N} f_i(t, s(t))$$

remain bounded. Then $\forall \varepsilon > 0\ \exists K > 0\ \forall k \geq K$: *Diffusive coupling results in*

$$\boxed{\limsup_{t \to \infty} |x_i(t) - x_j(t)| < \varepsilon \quad \forall i,j \in V}$$

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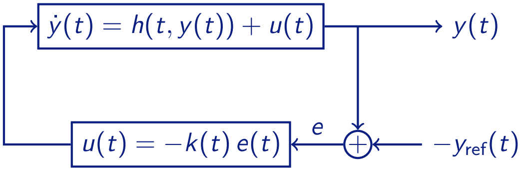
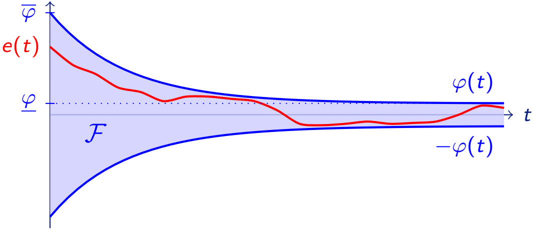
Reminder Funnel Controller





Theorem (Practical tracking, Ilchmann et al. 2002)

Funnel Control

$$k(t) = \frac{1}{\varphi(t) - |e(t)|}$$

works, in particular, errors remains within funnel for all times.

Funnel synchronization

Reminder diffusive coupling: $u_i = -k_i e_i$ with $e_i = x_i - \overline{x}_i$.

Combine diffusive coupling with Funnel Controller

$$u_i(t) = -k_i(t)\, e_i(t) \quad \text{with} \quad k_i(t) = \frac{1}{\varphi(t) - |e_i(t)|}$$

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Example (taken from Kim et al. 2015)

Simulations in the following for $N = 5$ agents with dynamics

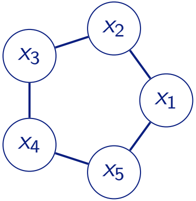
$$f_i(t, x_i) = (-1 + \delta_i)x_i + 10\sin t + 10m_i^1 \sin(0.1t + \theta_i^1) + 10m_i^2 \sin(10t + \theta_i^2),$$

with randomly chosen parameters $\delta_i, m_i^1, m_i^1 \in \mathbb{R}$ and $\theta_i^1, \theta_i^2 \in [0, 2\pi]$.

Parameters identical in all following simulations, in particular $\delta_2 > 1$, hence agent 2 has unstable dynamics (without coupling).

Simulation with constant k

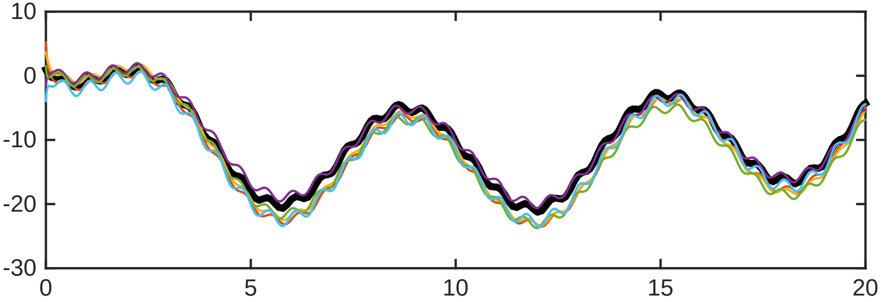
$u_i = -k\, e_i$ with $k = 10$



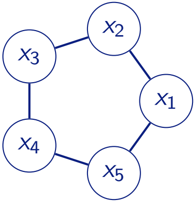
black curve:

$$\dot{s}(t) = \frac{1}{N}\sum_{i=1}^{N} f_i(t, s(t))$$

$$s(0) = \frac{1}{N}\sum_{i=1}^{N} x_i(0)$$



Funnel synchronization

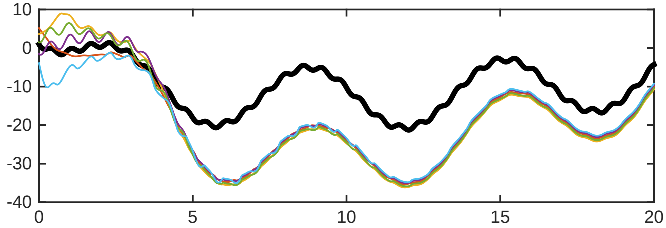


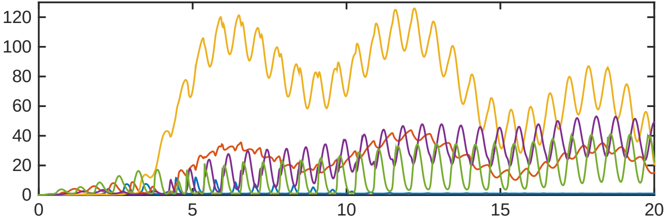
$$u_i(t) = -k_i(t)e_i(t)$$

$$k_i(t) = \frac{1}{\varphi(t) - |e_i(t)|}$$

$$\varphi(t) = \underline{\varphi} + (\overline{\varphi} - \underline{\varphi})e^{-\lambda t}$$

$$\overline{\varphi} = 20,\ \underline{\varphi} = 1,\ \lambda = 1$$





Observations for funnel synchronization from simulations

Funnel synchronization seems to work

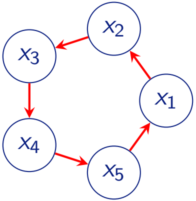
- errors remain within funnel
- practical synchronizations is achieved
- limit trajectory does not coincide with solution $s(\cdot)$ of

$$\dot{s}(t) = \frac{1}{N}\sum_{i=1}^{N} f_i(t, s(t)), \qquad s(0) = \frac{1}{N}\sum_{i=1}^{N} x_i.$$

What determines the new limiting trajectory?

- Coupling graph?
- Funnel shape?
- Gain function?

Funnel synchronization, directed graph

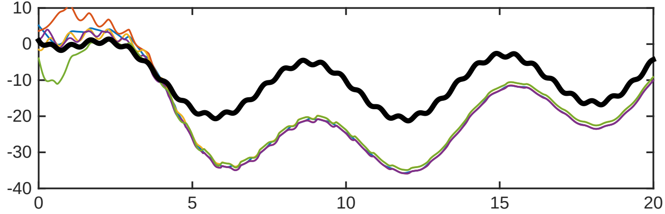


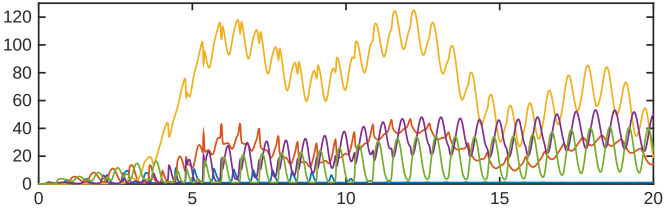
$$u_i(t) = -k_i(t)e_i(t)$$

$$k_i(t) = \frac{1}{\varphi(t) - |e_i(t)|}$$

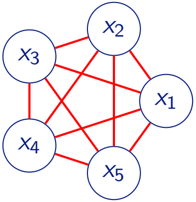
$$\varphi(t) = \underline{\varphi} + (\overline{\varphi} - \underline{\varphi})e^{-\lambda t}$$

$$\overline{\varphi} = 20,\ \underline{\varphi} = 1,\ \lambda = 1$$





Funnel synchronization, complete graph

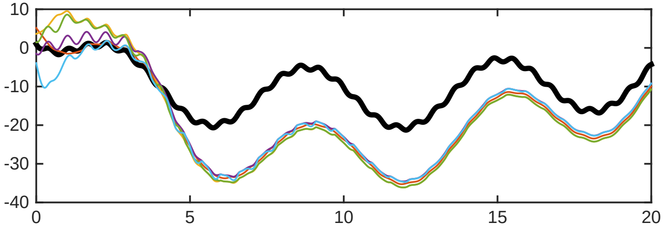


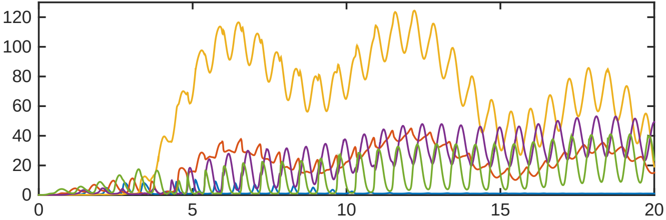
$$u_i(t) = -k_i(t)e_i(t)$$

$$k_i(t) = \frac{1}{\varphi(t) - |e_i(t)|}$$

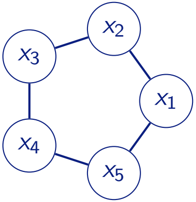
$$\varphi(t) = \underline{\varphi} + (\overline{\varphi} - \underline{\varphi})e^{-\lambda t}$$

$$\overline{\varphi} = 20,\ \underline{\varphi} = 1,\ \lambda = 1$$





Funnel synchronization with bigger funnel

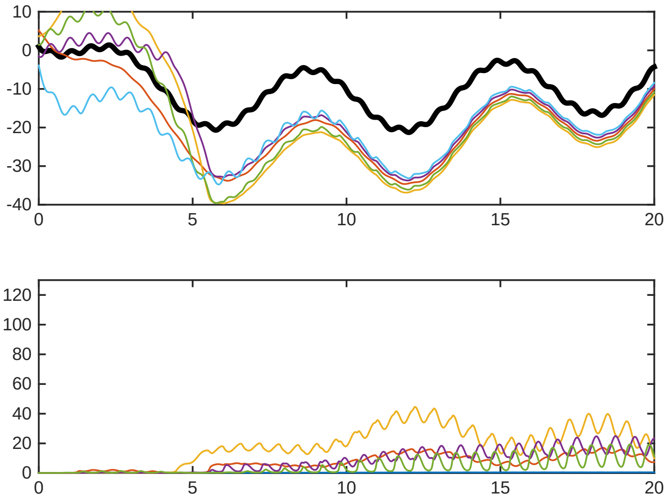


$$u_i(t) = -k_i(t)e_i(t)$$

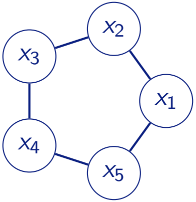
$$k_i(t) = \frac{1}{\varphi(t) - |e_i(t)|}$$

$$\varphi(t) = \underline{\varphi} + (\overline{\varphi} - \underline{\varphi})e^{-\lambda t}$$

$\overline{\varphi} = 30$, $\underline{\varphi} = 2$, $\lambda = 0.3$



Funnel synchronization with additional amplification

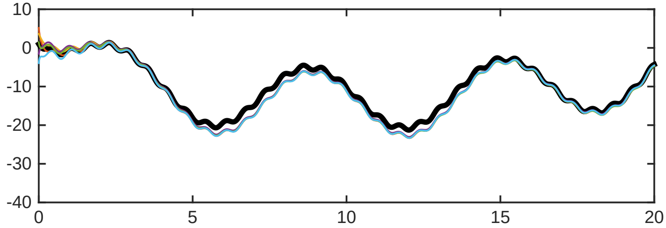


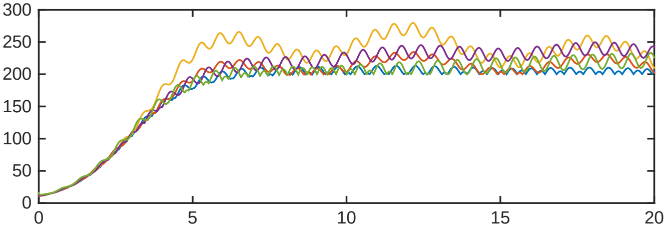
$$u_i(t) = -k_i(t)e_i(t)$$

$$k_i(t) = \frac{200}{\varphi(t) - |e_i(t)|}$$

$$\varphi(t) = \underline{\varphi} + (\overline{\varphi} - \underline{\varphi})e^{-\lambda t}$$

$$\overline{\varphi} = 20,\ \underline{\varphi} = 1,\ \lambda = 1$$





Content

Decentralized and weakly centralized Funnel synchronization

For fully decentralized Funnel synchronization

$$u_i(t) = -k_i(t)e_i(t) \quad \text{with} \quad k_i(t) = \frac{1}{\varphi(t) - |e_i(t)|}$$

no theoretical results available yet.

Weakly centralized Funnel synchronization

Analogously as for diffusive coupling, all agents use the same gain:

$$u_i(t) = -k_{\max}(t)\, d_i\, e_i(t) \quad \text{with} \quad k_{\max}(t) := \max_{i \in V} \frac{1}{\varphi(t) - |e_i(t)|}$$

First theoretical result

Theorem

Assumption:

- *No „finite escape time" of x_i*
- *The graph is connected, undirected and d-regular with*

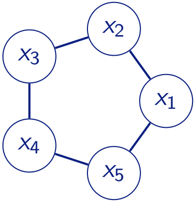
$$d > \frac{N}{2} - 1$$

- *Funnel boundary $\varphi : [0, \infty) \to [\underline{\varphi}, \overline{\varphi}]$ is differentiable, non-increasing and*

$$|e_i(0)| < \varphi(0), \quad \forall i = 1, 2, \dots, N.$$

Then weakly centralized funnel synchronization works.

Simulation weakly centralized Funnel synchronization



$$u_i(t) = -k_{\max}(t) e_i(t)$$

$$k_{\max}(t) = \max_{i \in V} \frac{1}{\varphi(t) - |e_i(t)|}$$

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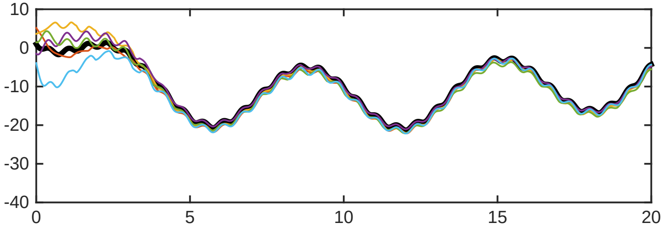

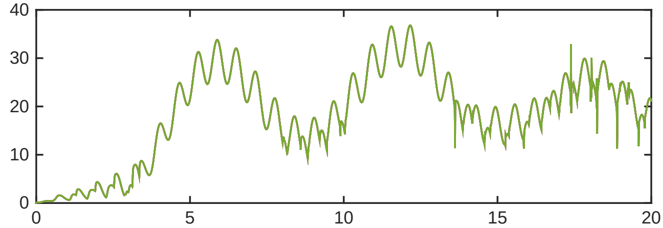

Summary

Combining diffusive coupling with funnel control leads to funnel synchronization

- local error feedback
- time-varying gain
- guaranteed transient behavior
- simulations look promising
- theoretical proof for weakly centralized funnel synchronization

Open questions

- limit trajectory
- weakly centralized case: non-regular graph or d small
- decentralized case