The bang-bang funnel controller: time delays and case study

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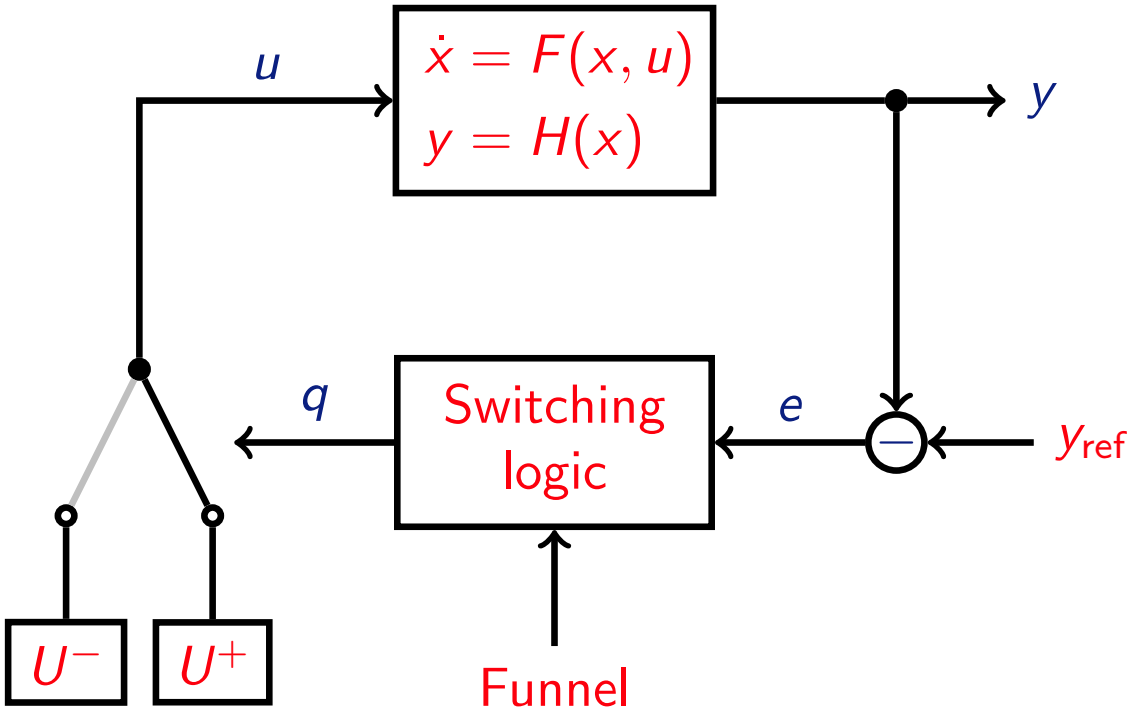
Content

Control setup



Goal: **Tracking** with prespecified **error bounds** for **uncertain system** with only **two control values**

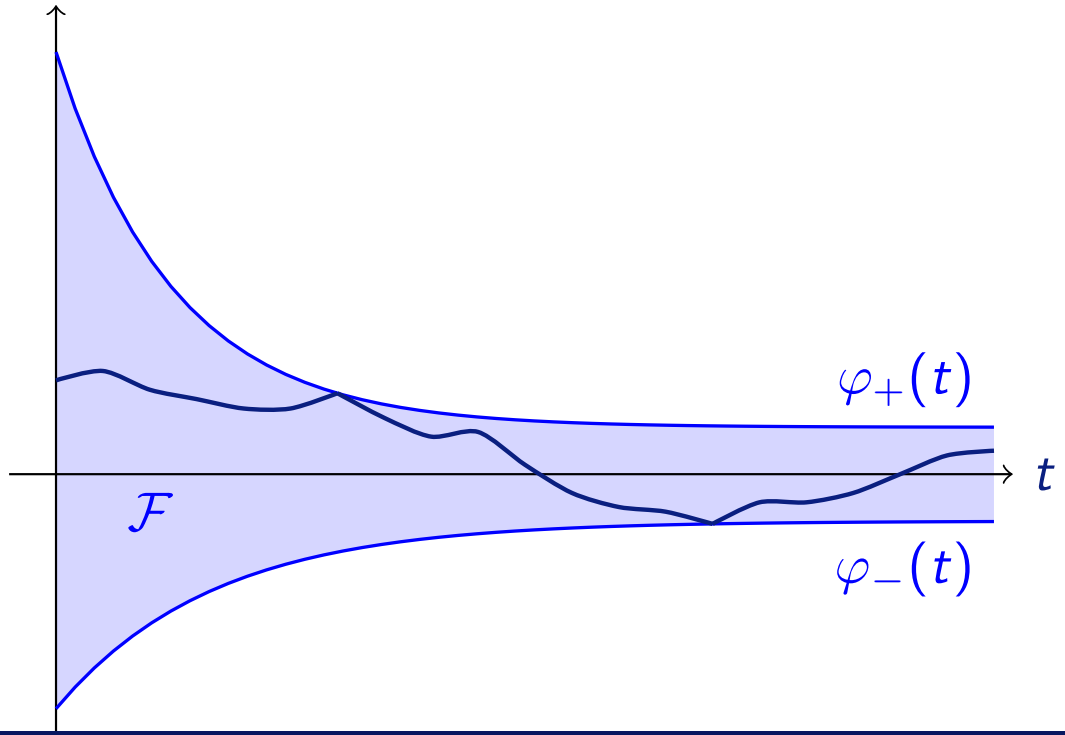
The funnel

Control objective

Error $e := y - y_{\text{ref}}$ evolves within *funnel*

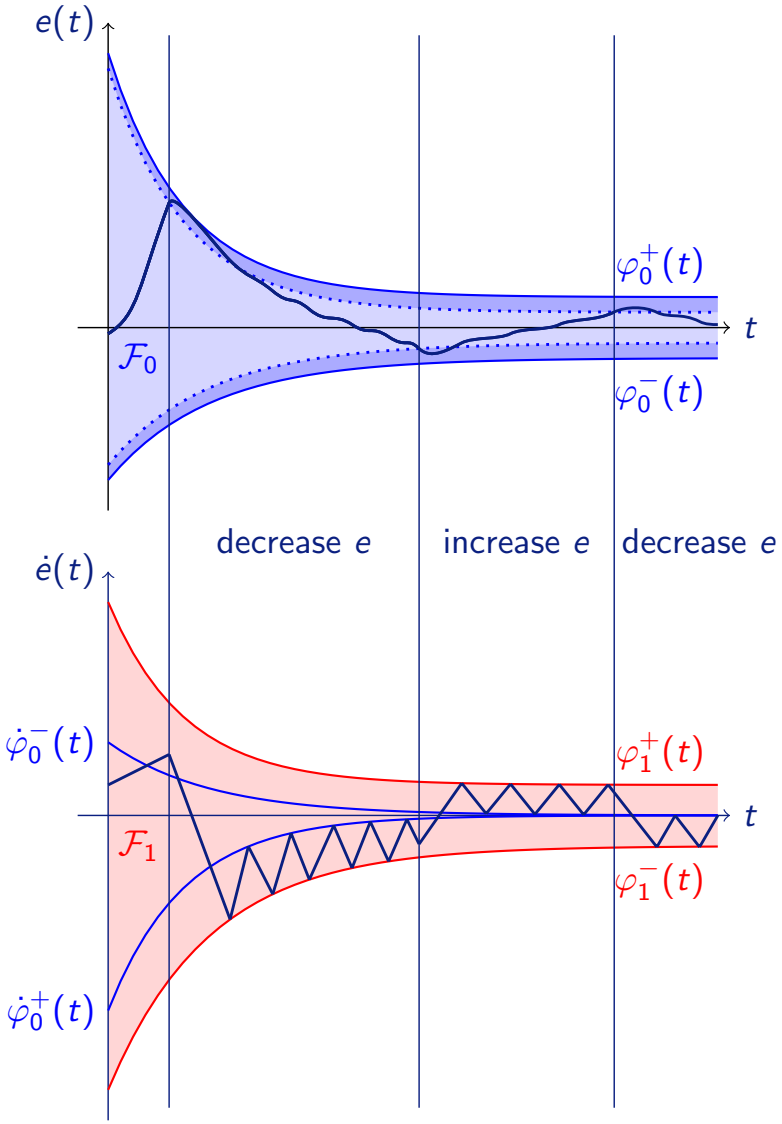
$$\mathcal{F} = \mathcal{F}(\varphi_-, \varphi_+) := \{ (t, e) \mid \varphi_-(t) \leq e \leq \varphi_+(t) \}$$

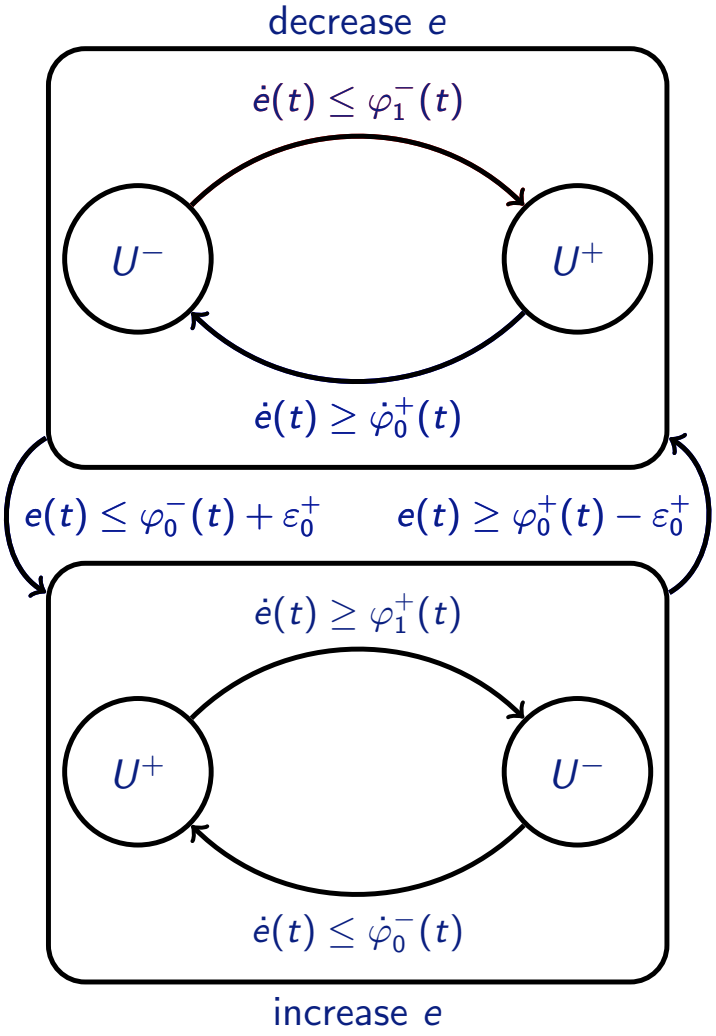
where $\varphi_\pm : \mathbb{R}_{\geq 0} \to \mathbb{R}_{>0}$



- time-varying strict error bound
- transient behaviour
- practical tracking ($|e(t)| < \lambda$ for $t >> 0$)
- proposed by Ilchmann et al. 2002

The switching logic (CDC 2010)





Theoretical result

Structural assumption and feasibility

- Relative degree two:

$$u(t) << 0 \quad \Rightarrow \quad \ddot{y}(t) << 0$$
$$u(t) >> 0 \quad \Rightarrow \quad \ddot{y}(t) >> 0$$

- feasibility of funnels
- input values large enough

Theorem (CDC 2010)

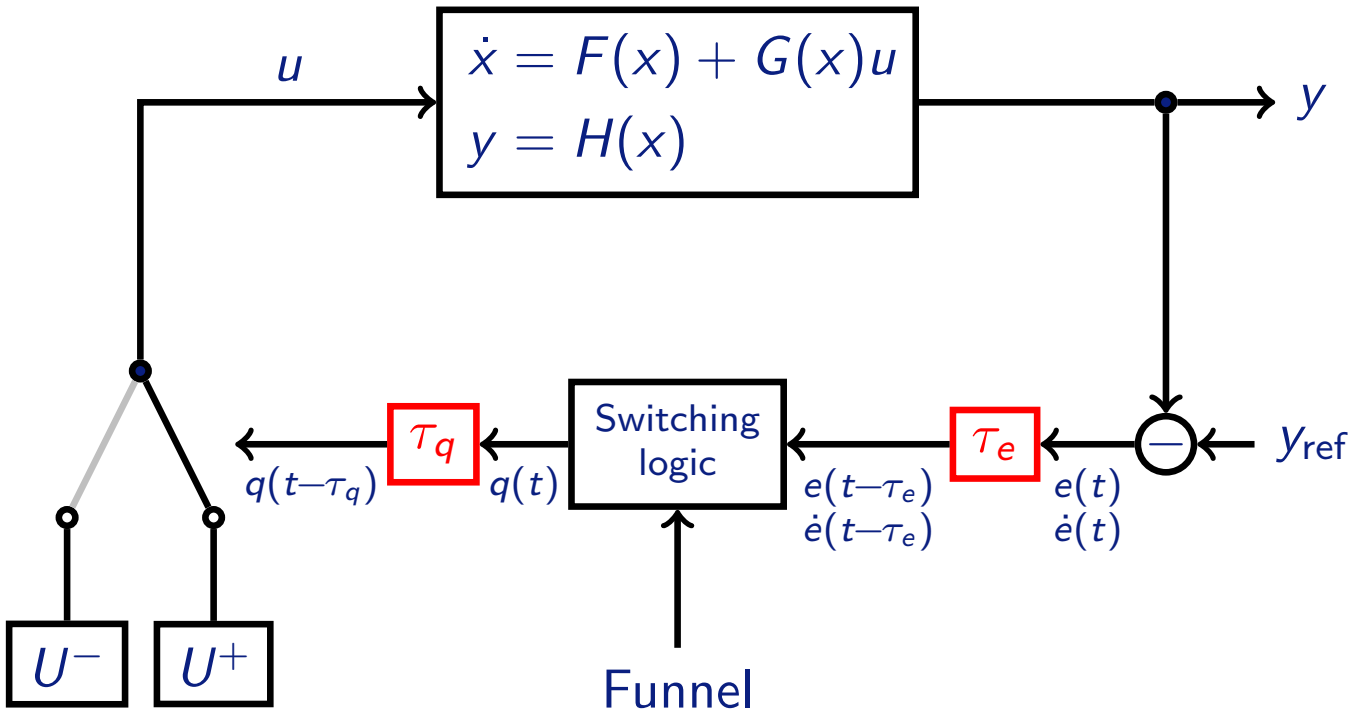
Relative degree two & Funnels & simple switching logic & Feasibility

$\Rightarrow$

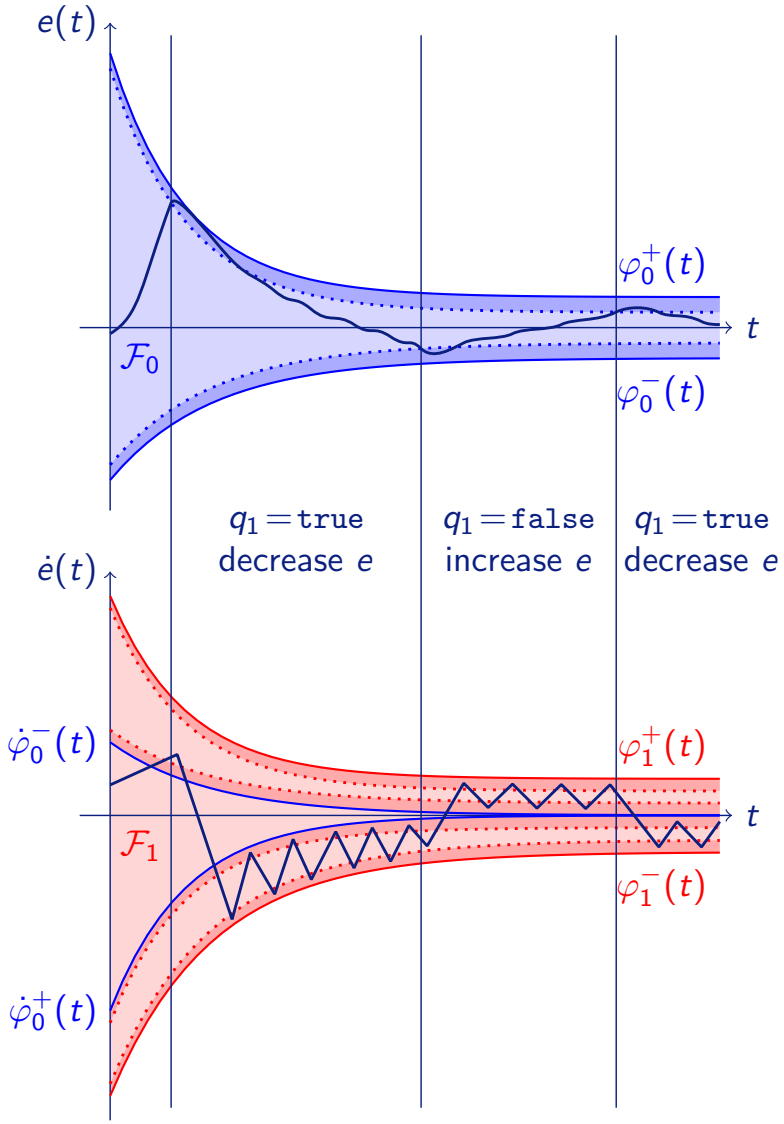
Bang-bang funnel controller works:

- *existence and uniqueness of global solution*
- *error and its derivative remain within funnels for all time*
- *no zeno behaviour*

More realistic scenario



Adjusted switching logic and new feasibility assumption



Same switching logic

Apart from introduction of safety distance $\varepsilon_1^{\pm}$ also for the derivative funnel

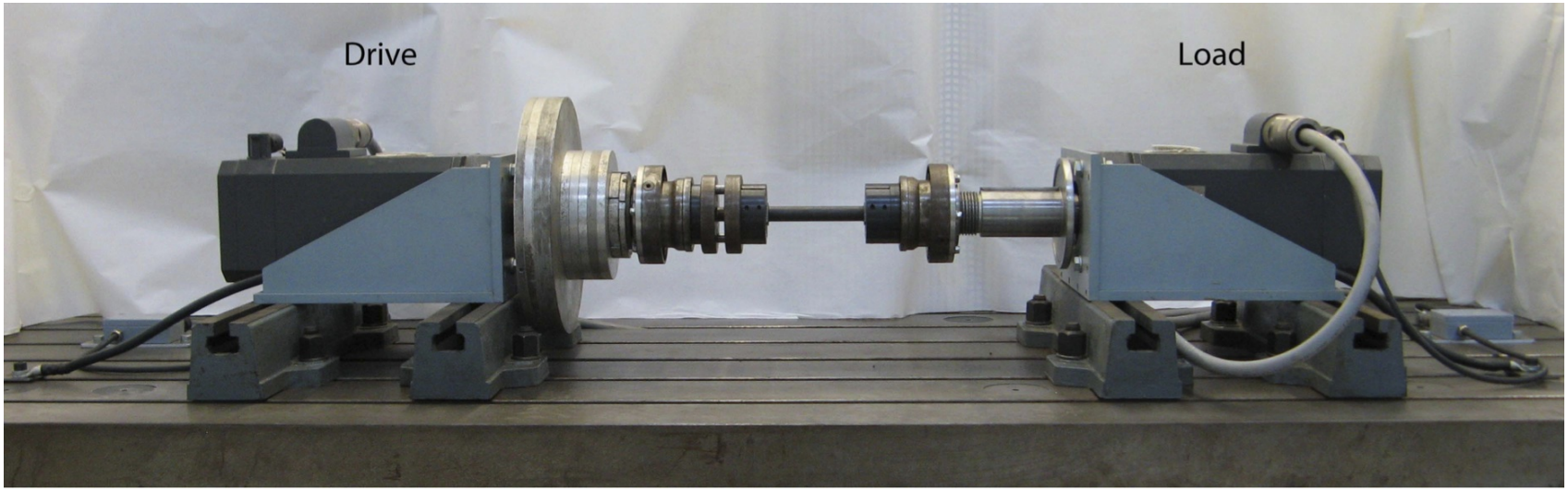
New feasibility assumption

Bounding the time delay $\tau_e + \tau_q$ in terms of safety distances $\varepsilon_1^{\pm}$ and $\varepsilon_0^{\pm}$.

Theorem

Bang-bang funnel controller also works in the presence of sufficiently small time delays.

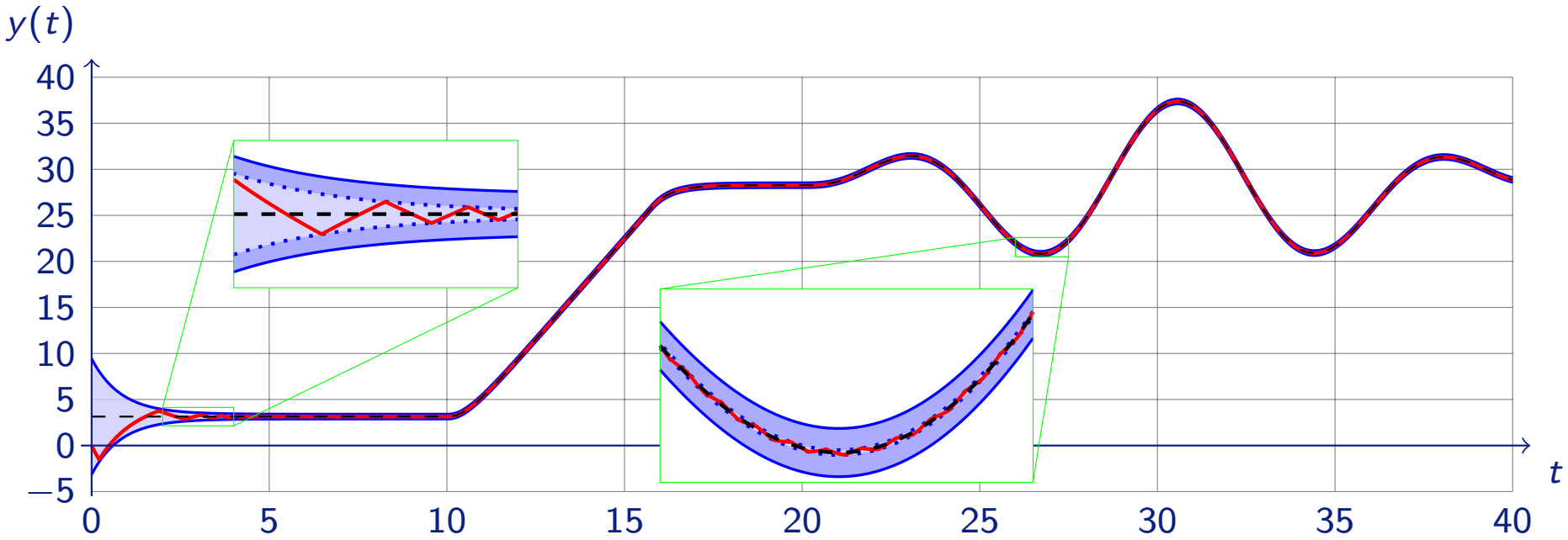
Physical background for simulation



$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \gamma \end{bmatrix} \big(u(t) + u_L(t) - (Tx_2)(t)\big),$$
$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t),$$

x_1: angle of the rotary machine
$x_2 = \dot{x}_1$: angular velocity
u_L: unknown load torque
$T : \mathcal{C}(\mathbb{R}_{\geq 0} \to \mathbb{R}) \to \mathcal{L}^\infty_{\text{loc}}(\mathbb{R}_p \to \mathbb{R})$ friction operator

Tracking of given reference trajectory



Feasibility conditions too conservative

- simulation carried out with $U^{\pm} = \pm 2425 Nm$
- much larger than technical possible ($\pm 22 Nm$)
- switching frequency (about $10^4 Hz$) too high

Heuristic improvement

Underlying problem

- good long-time accuracy $\Rightarrow$ small safety distance
- large error-tolerance $\Rightarrow$ need large safety distance

Use time-varying safety distances

- works very well in simulations
- switching logic remains the same
- formal proof even more technical and not carried out yet

Summary

- Introduced new controller design: Bang-bang funnel controller
  - Design only depends on relative degree
  - extremely simple
- Feasibility assumptions
  - U_+, U_- must be large enough
  - in terms of bounds on systems dynamics
  - better performance $\Rightarrow$ larger values for U_+, U_-
- Tolerates time delays
- Higher relative degree (not presented here)
  - Switching logic remains simple (hierarchically)
  - Feasibility assumptions remain similar
  - Switching frequency increase significantly
  - for details see: Liberzon & Trenn, IEEE TAC 2013 (to appear)