The bang-bang funnel controller: An experimental verification

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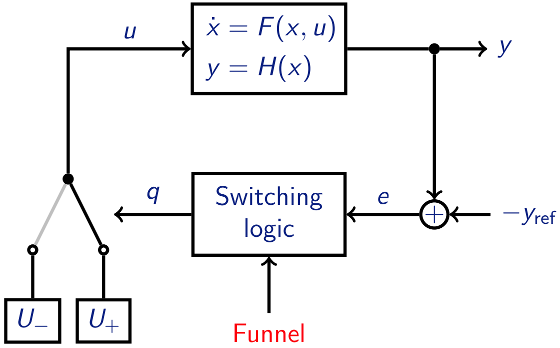
Content

The Bang-Bang funnel controller: Basic idea



Here: System has relative-degree two with bounded zero dynamics

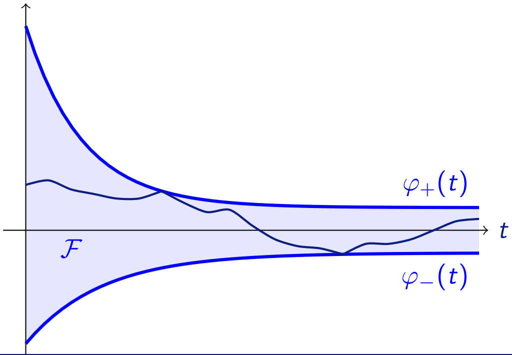
The funnel

Control objective

Error $e := y - y_{\text{ref}}$ evolves within *funnel*

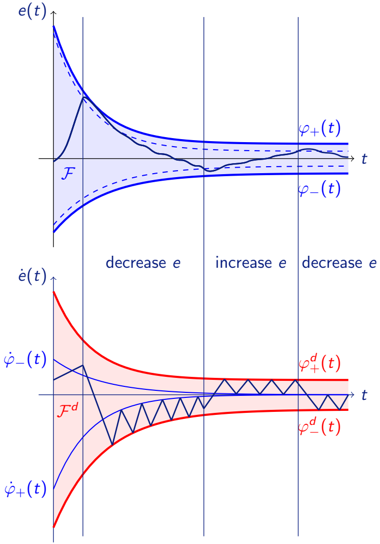
$$\mathcal{F} = \mathcal{F}(\varphi_-, \varphi_+) := \{ \ (t, e) \ | \ \varphi_-(t) \leq e \leq \varphi_+(t) \ \}$$

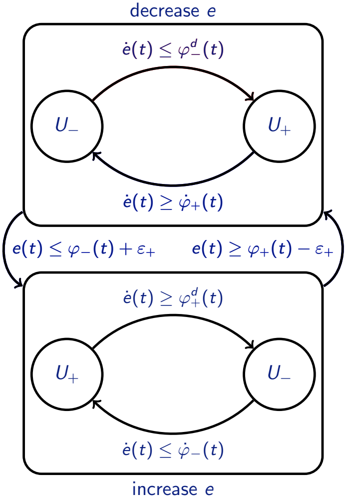
where $\varphi_\pm : \mathbb{R}_{\geq 0} \to \mathbb{R}_{>0}$



- time-varying strict error bound
- transient behaviour
- practical tracking ($|e(t)| < \lambda$ for $t >> 0$)

The switching logic (Liberzon & T. 2010)





Theoretical result relative degree two

Feasibility

- feasibility of funnels
- input values large enough

Theorem (Bang-bang funnel controller)

Relative degree two & Funnels & simple switching logic & Feasibility

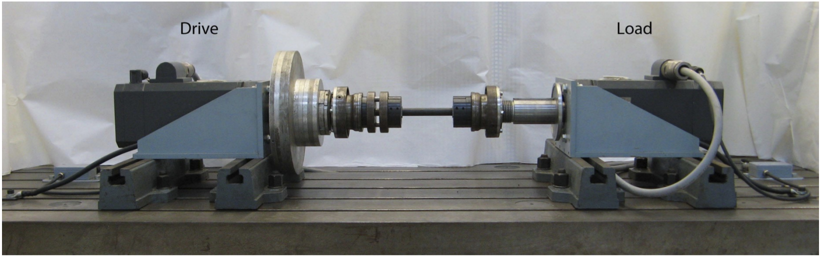
$\Rightarrow$

Bang-bang funnel controller works:

- *existence and uniqueness of global solution*
- *error and its derivative remain within funnels for all time*
- *no zeno behaviour*

Content

Experimental setup



$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \gamma \end{bmatrix} \big(u(t) + u_L(t) - (Tx_2)(t) \big),$$
$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t),$$

x_1: angle of the rotary machine
$x_2 = \dot{x}_1$: angular velocity
u_L: unknown load torque
$T : \mathcal{C}(\mathbb{R}_{\geq 0} \to \mathbb{R}) \to \mathcal{L}^\infty_{\text{loc}}(\mathbb{R}_p \to \mathbb{R})$ friction operator

Control input limitations

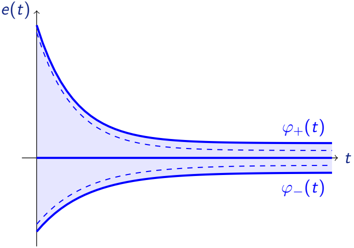
Control inputs

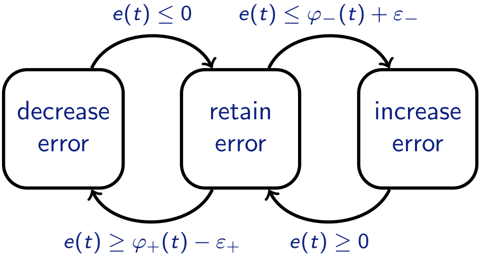
$$U_+ = 22\,\mathrm{Nm} \quad U_0 = 0\,\mathrm{Nm} \quad U_- = -22\,\mathrm{Nm}$$

Problem and solution

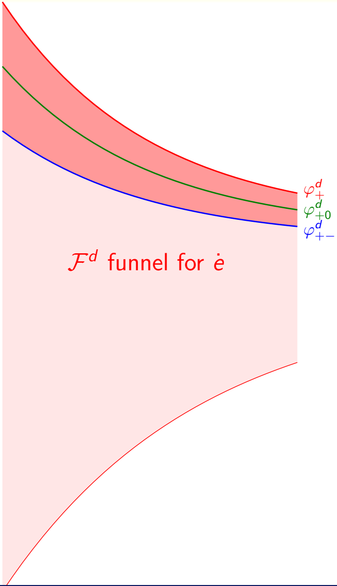
Direct switch from U_+ to U_- and vice versa not "healthy" for machine

$\Rightarrow$ Definition of new switching logic necessary!



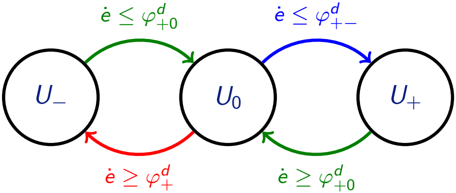


New switching logic

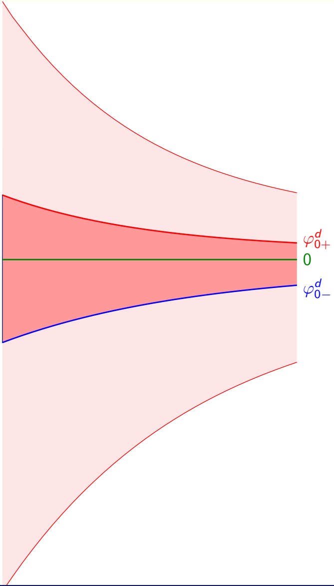


increase error

$\Rightarrow$ make and keep $\dot{e}(t)$ positive

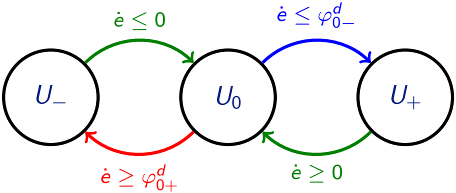


New switching logic

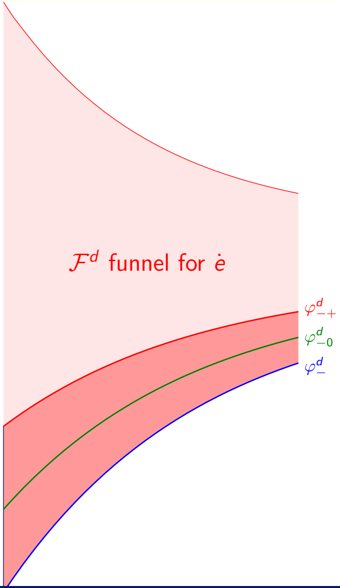


retain error

$\Rightarrow$ make and keep $\dot{e}(t)$ close to zero

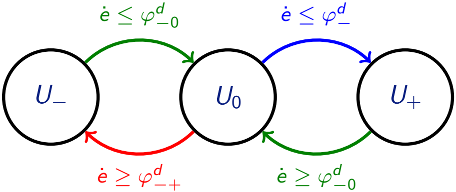


New switching logic



decrease error

$\Rightarrow$ make and keep $\dot{e}(t)$ negative



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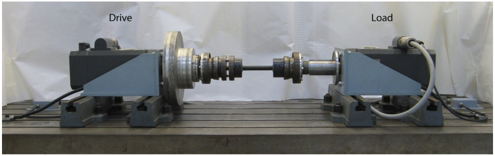
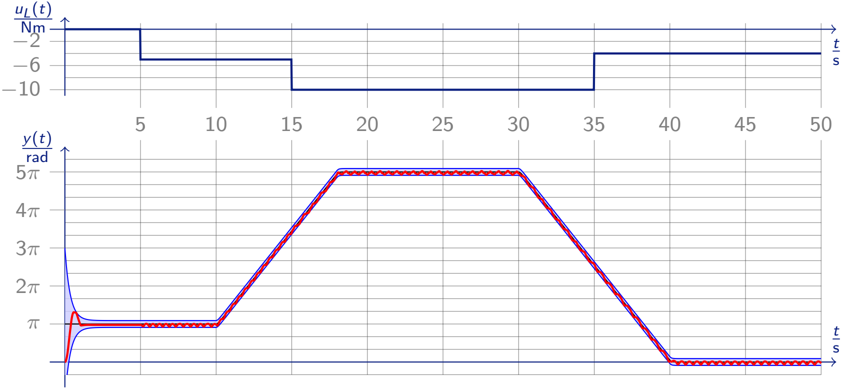
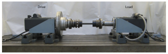
Reference tracking in the presence of unknown load





Transient response without load disturbance



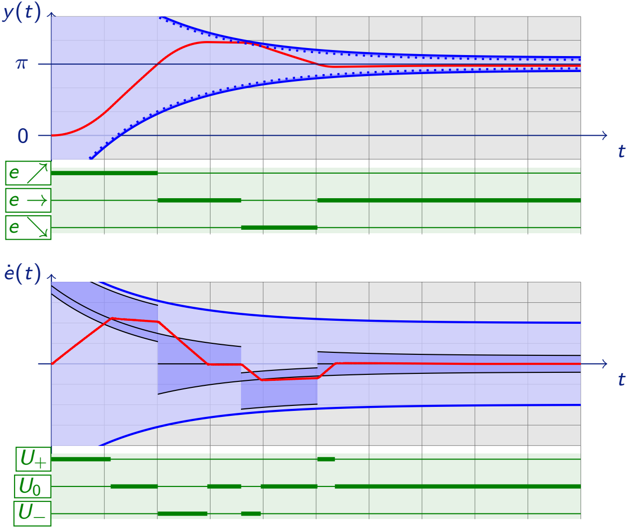
$t \in [0, 2s]$

$u_L \equiv 0$

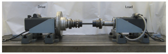
$y_{\text{ref}} \equiv \pi$

$y(0) = 0$

$\dot{y}(0) = 0$



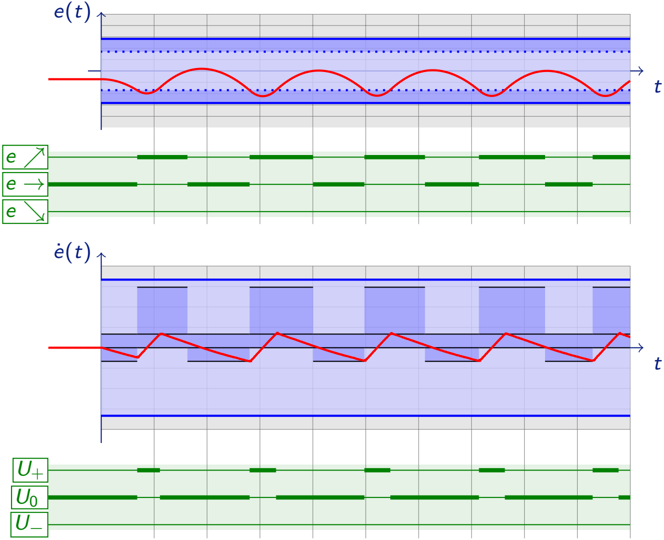
Response in the presence of load disturbance



$t \in [5s, 7s]$

$u_L \equiv -4\,\mathrm{Nm}$

$y_{\text{ref}} \equiv \pi$



Summary