Analogue Implementation of the Funnel Controller

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Berlin, March 28th 2006
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Scope of funnel control

\[ u(t) = -k(t)e(t) \]

Aim
Tracking of a reference signal.
Scope of funnel control

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Properties of the system class
- nonlinear functional differential equations
- includes functional effects like hysterises and delays
- high-gain stabilizable

System

\[ u(t) = -k(t)e(t) \]

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Control objectives

- **Practical asymptotic stability** of the error, i.e. for a given \( \lambda > 0 \)

  \[ \exists T > 0 \quad \forall t \geq T : \quad |e(t)| < \lambda. \]

- **Prescribed transient behaviour**, e.g. guaranteeing an upper bound for the overshoot or a prescribed transient time.

- **Independence of system parameters**, i.e. the same controller works for all systems of the systems class.
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Control objectives $\iff$ Prescribed funnel

The funnel $\mathcal{F} \subseteq \mathbb{R}_{\geq 0} \times \mathbb{R}^n$: $e(0)$, $e(t)$
The control law:

\[ u(t) = -k(t) e(t) \]

The gain function

\[ k(t) = K_F(t, e(t)) \]

\[ K_F : \mathcal{F} \rightarrow \mathbb{R}_{\geq 0} \]
Theoretical results

Necessary condition on the gain function $K_F$

1. The closer the error to the funnel boundary, the larger the gain.

2. If the error is away from the funnel boundary then the gain is not unnecessarily large.
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1. The closer the error to the funnel boundary, the larger the gain.
2. If the error is away from the funnel boundary then the gain is not unnecessarily large.

Theorem

The funnel controller $u(t) = -K_F(t, e(t)) e(t)$ achieves the control objectives, i.e. ensures that the errors evolves within the prespecified funnel independently of the system’s parameters.

Further results

- **First funnel controller**
  Ilchmann, Ryan, Sangwin (2002): *Tracking with prescribed transient behaviour*

- **Application to a model of chemical reactors**
  Ilchmann, Trenn (2004): *Input constrained funnel control with applications to chemical reactor models*

- **Higher relative degree systems**
  Ilchmann, Ryan, Townsend (2006): *Tracking with prescribed transient behaviour for nonlinear systems of known relative degree*
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Now to Nagendra ...