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Adaptive tracking within prescribed funnels

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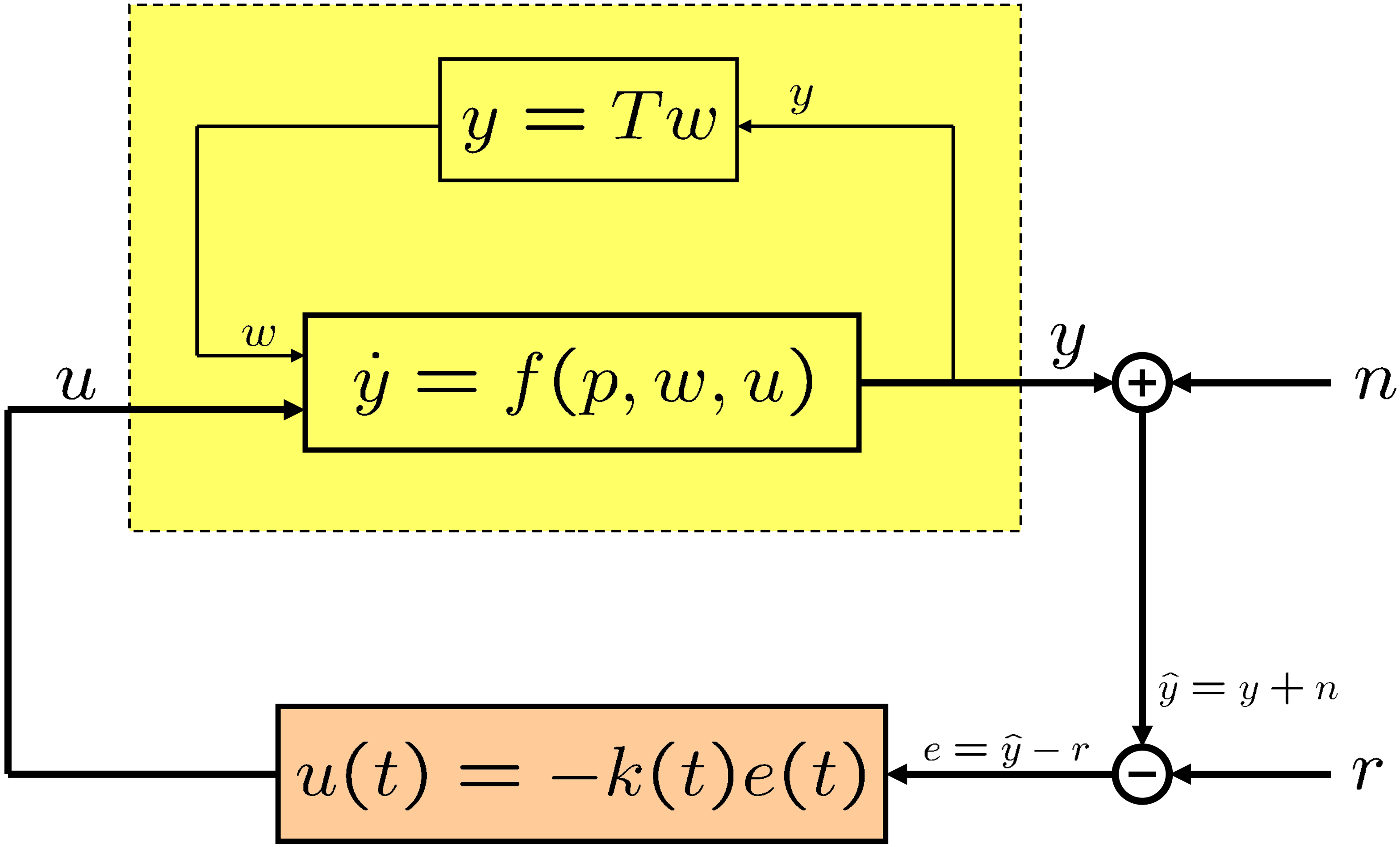
Aim: output tracking of nonlinear system by proportional error feedback



Class of systems: Linear SISO example

$$\dot{y}(t) = A_1\ y(t) + A_2\ z(t) + cb\ u(t) \qquad y(0) = y^0 \in \mathbb{R}$$
$$\dot{z}(t) = A_3\ y(t) + A_4\ z(t) \qquad z(0) = z^0 \in \mathbb{R}^{n-1}$$

$cb > 0$ $\qquad$ A_4 Hurwitz

$\Rightarrow$

$u(t) = -k\ y(t)$ stabilises for $k \gg 1$

System class

$$\dot{y}(t) = f\Big(p(t), \big(Ty\big)(t), u(t)\Big) \qquad y\Big|_{[-h,0]} = y^0 \in C\big([-h,0], \mathbb{R}^M\big)$$

1. $p \in L^\infty\big(\mathbb{R}_{\geq 0}, \mathbb{R}^P\big)$
2. $f \in C\big(\mathbb{R}^P \times \mathbb{R}^Q \times \mathbb{R}^M, \mathbb{R}^M\big)$
3. $\forall$ compact $\mathcal{C} \subseteq \mathbb{R}^P \times \mathbb{R}^Q \quad \forall (u_n) \in \big(\mathbb{R}^M \setminus \{0\}\big)^{\mathbb{N}}$:

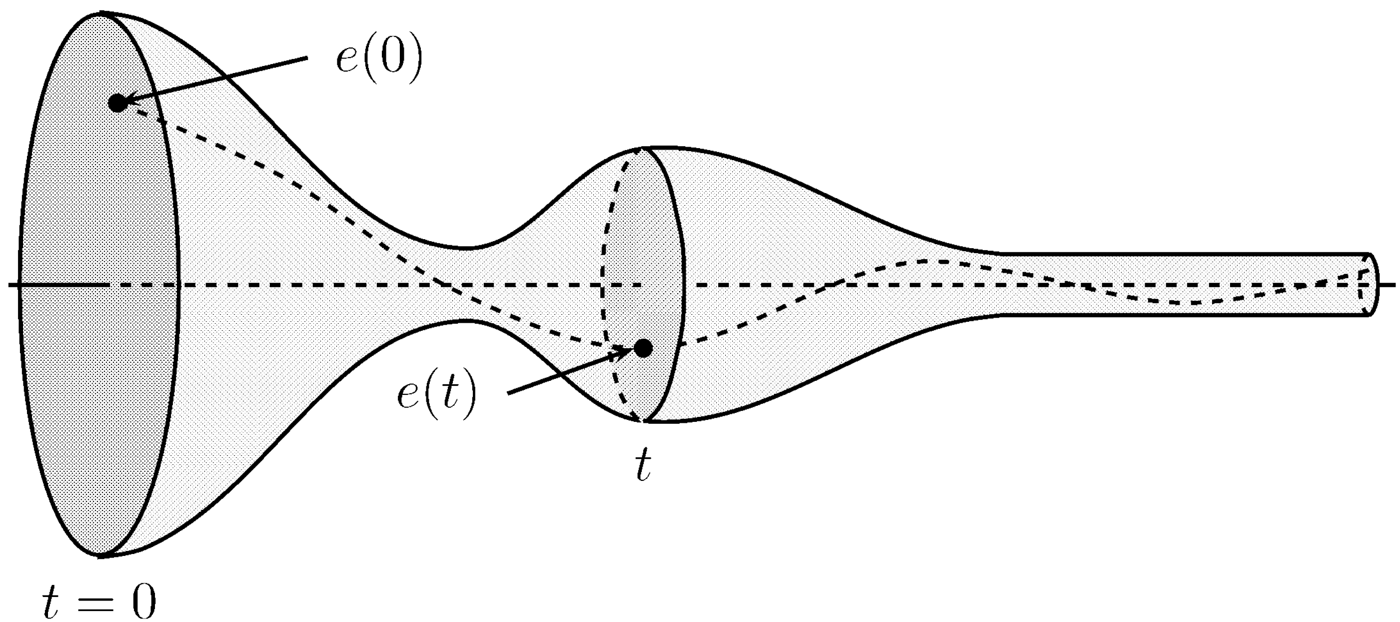
   $$\|u_n\| \to \infty \text{ as } n \to \infty \qquad \Rightarrow \qquad \lim_{n\to\infty} \min_{(v,w)\in\mathcal{C}} \langle u_n, f(v,w,u_n)\rangle / \|u_n\| = \infty$$

4. bounded, causal and locally Lipschitz operator
   $T : C\big([-h,\infty), \mathbb{R}^M\big) \to L^\infty_{\mathrm{loc}}(\mathbb{R}_{\geq 0}, \mathbb{R}^Q)$

Control objectives

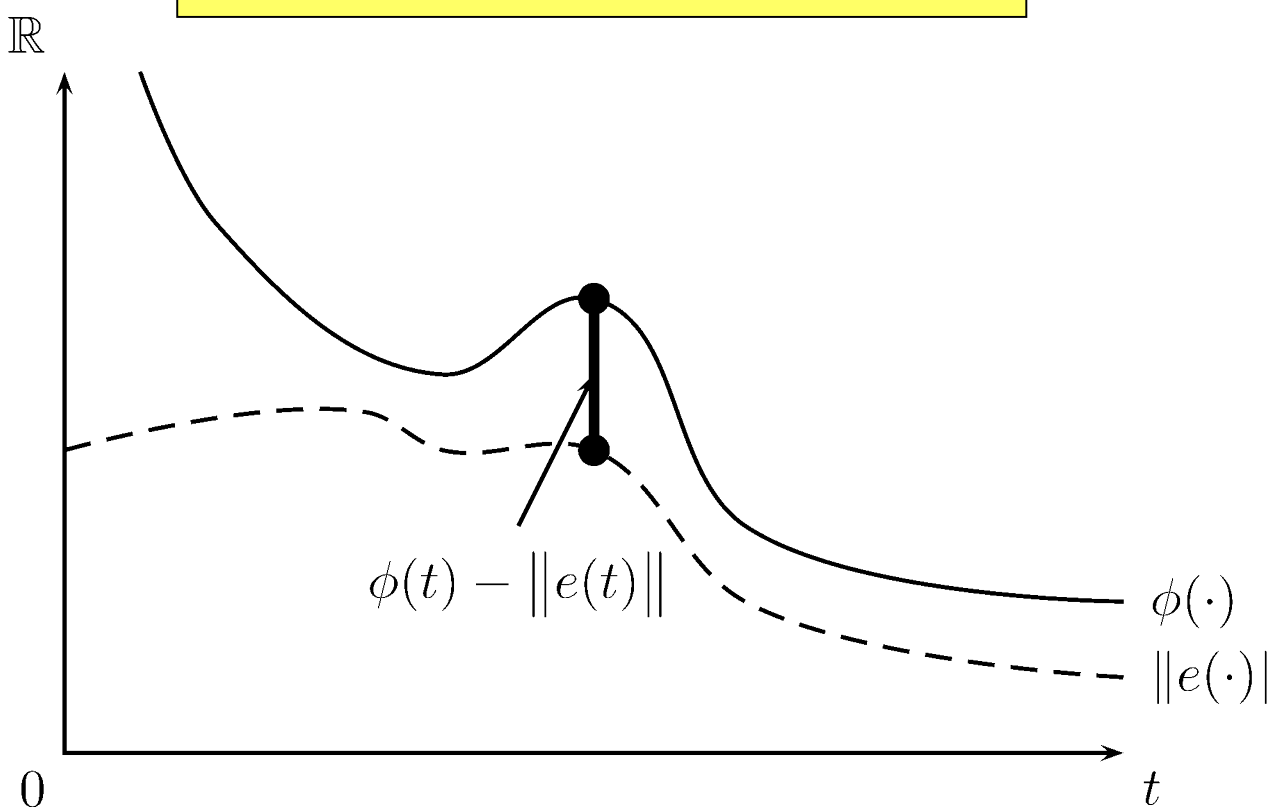
- λ -tracking
- prescribed transient behaviour
- a *low* high-gain

funnel $\mathcal{F}$



Control input $u(t) = -k(t)e(t)$

$$k(t) = \frac{1}{\phi(t) - \|e(t)\|}$$

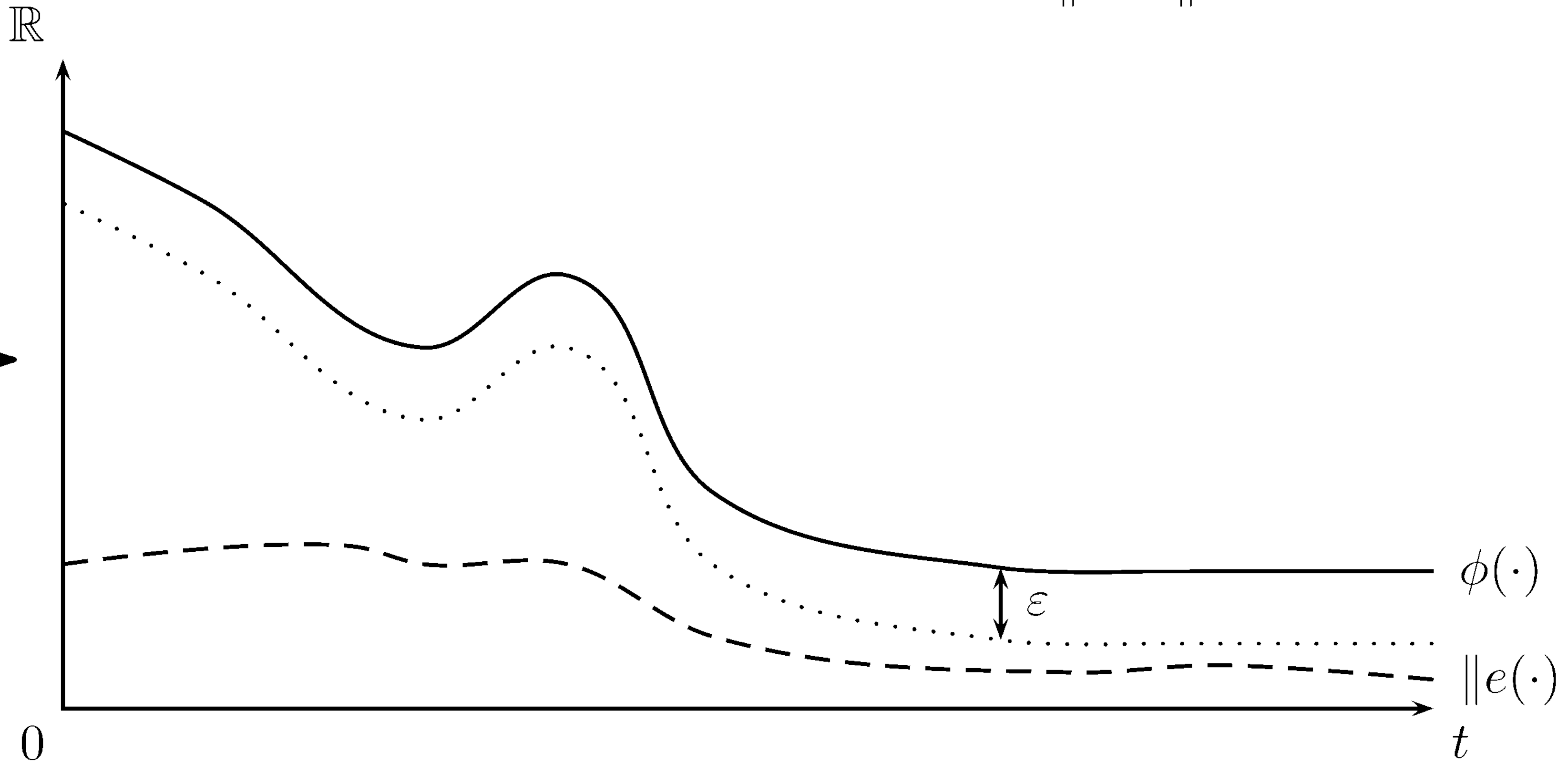


Theorem (Linear example)

$$\dot{y}(t) = A_1\ y(t) + A_2\ z(t) + cb\ u(t) \qquad y(0) = y^0 \in \mathbb{R}$$
$$\dot{z}(t) = A_3\ y(t) + A_4\ z(t) \qquad z(0) = z^0 \in \mathbb{R}^n$$

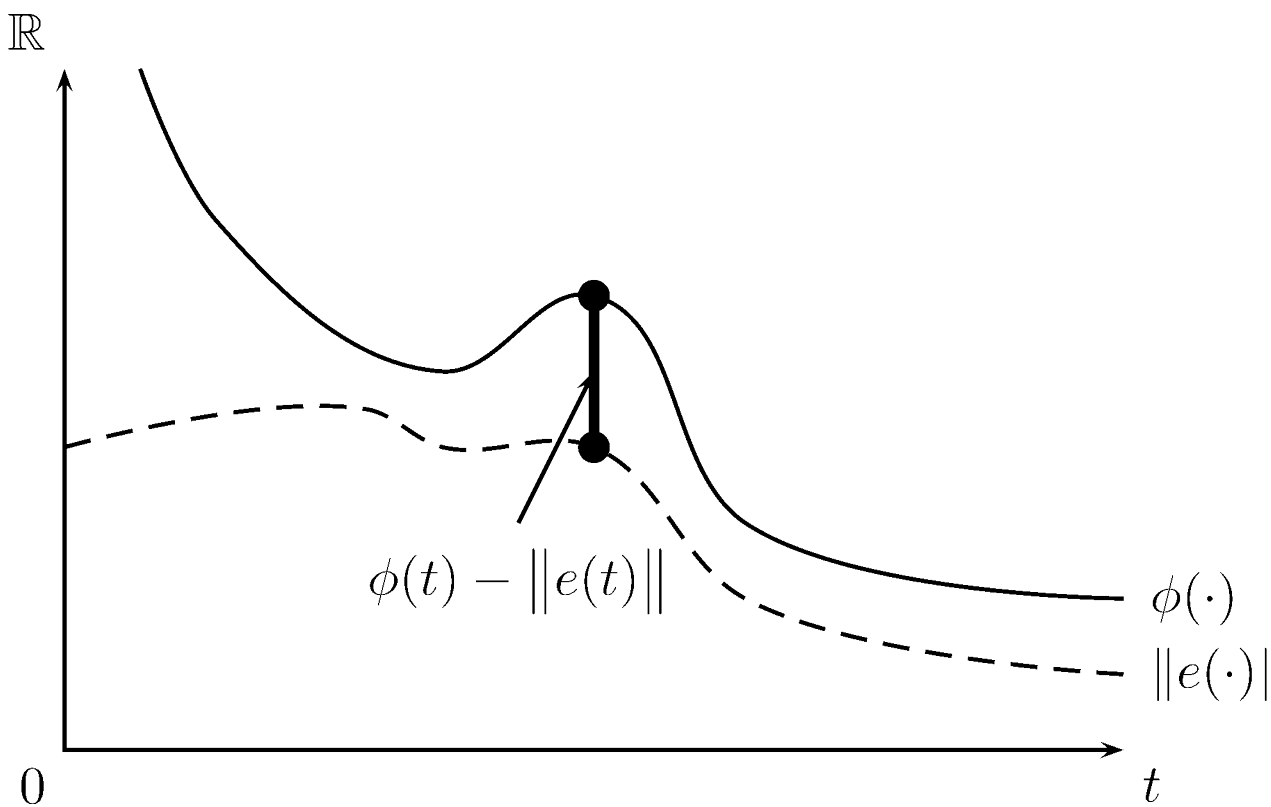
$$cb > 0 \qquad A_4 \text{ Hurwitz} \qquad u(t) = -\frac{1}{\phi(t) - \|e(t)\|}\, e(t)$$

$\Rightarrow$



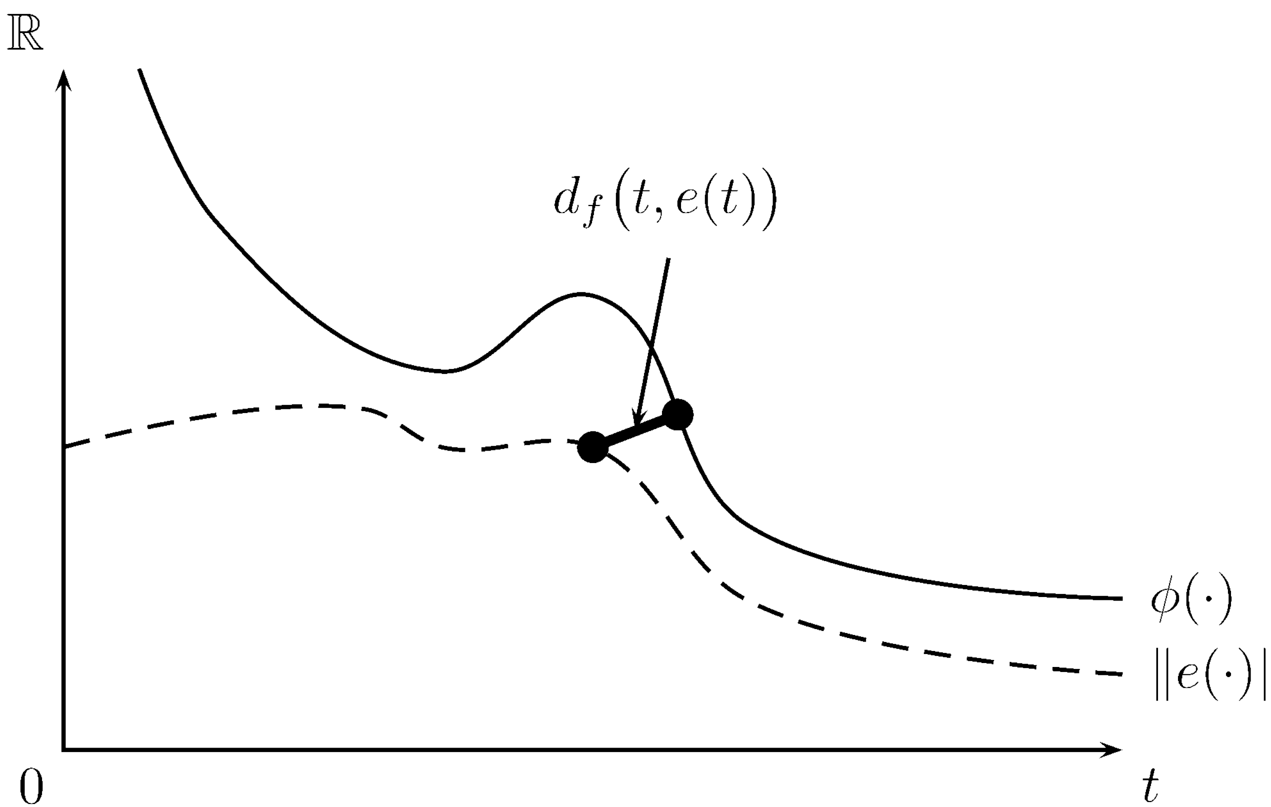
Control input $$u(t) = -k(t)e(t)$$

$$k(t) = \frac{1}{\phi(t) - \big\|e(t)\big\|} = K_{\mathcal{F}}\big(t, e(t)\big)$$



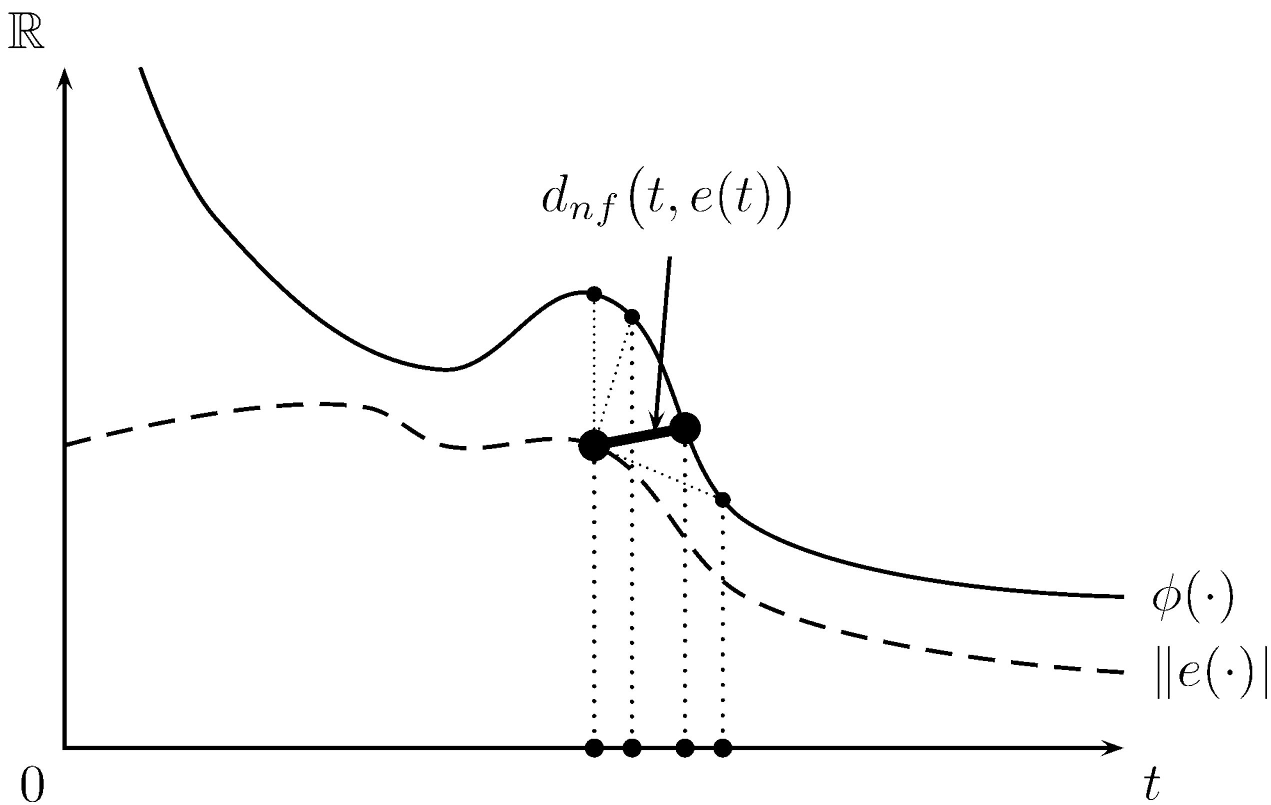
Control input $u(t) = -k(t)e(t)$

$$k(t) = \frac{1}{d_f\big(t, e(t)\big)} = K_{\mathcal{F}}\big(t, e(t)\big)$$

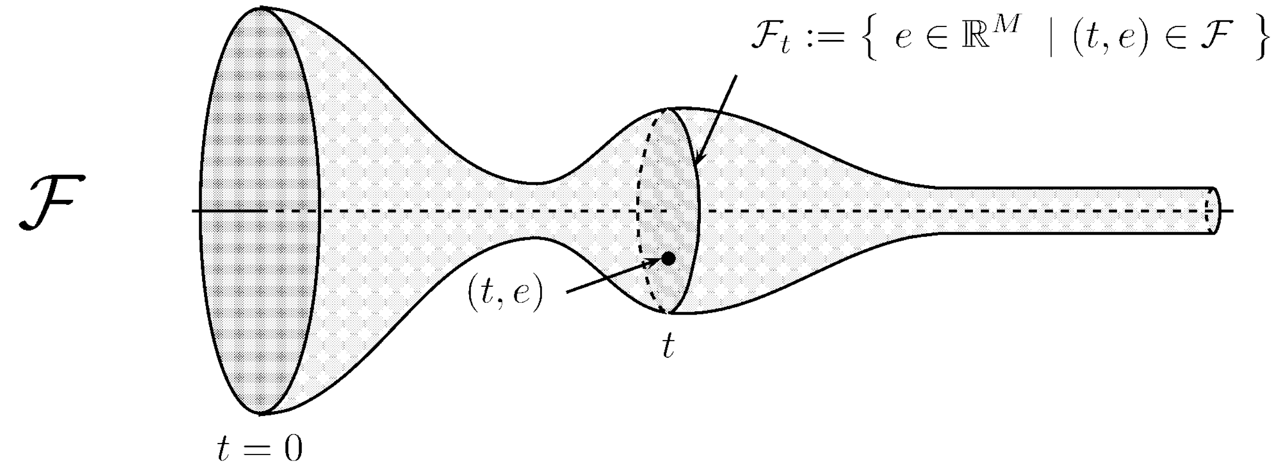


Control input $u(t) = -k(t)e(t)$

$$k(t) = \frac{1}{d_{nf}\big(t, e(t)\big)} = K_{\mathcal{F}}\big(t, e(t)\big)$$



Requisite properties of the gain function $K_{\mathcal{F}}$



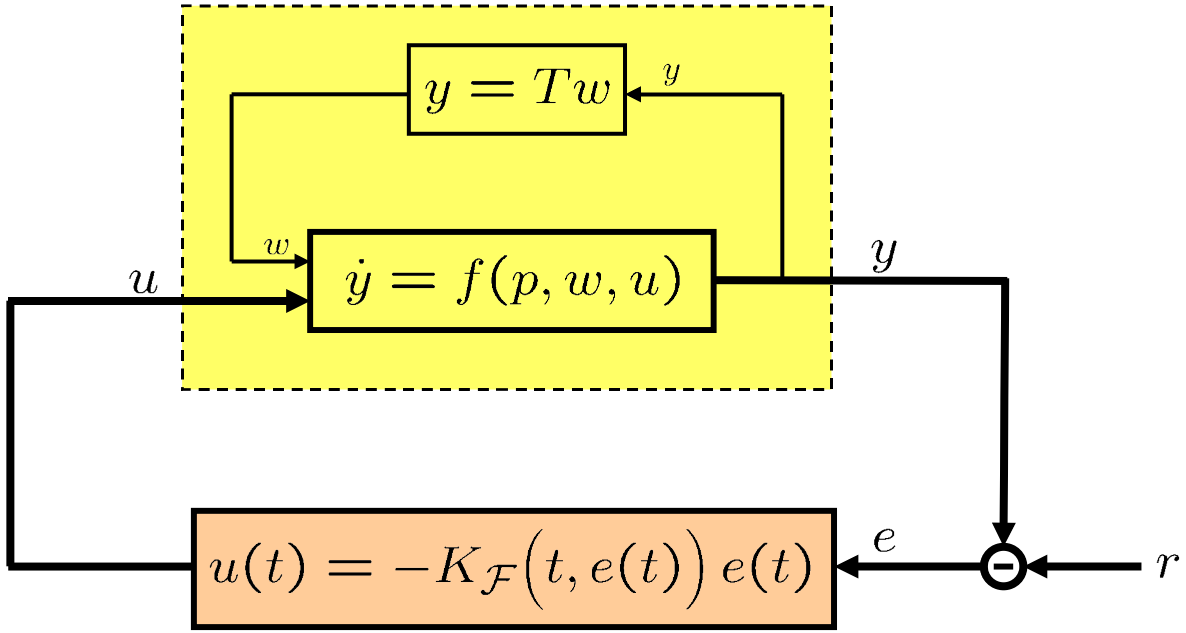
Property 1 $\quad \forall\, K > 0\ \exists\, \varepsilon > 0\ \forall\, (t,e) \in \mathcal{F}$:

$$\operatorname{dist}\big(e, \partial\mathcal{F}_t\big) \le \varepsilon \quad \Rightarrow \quad K_{\mathcal{F}}(t,e) \ge K$$

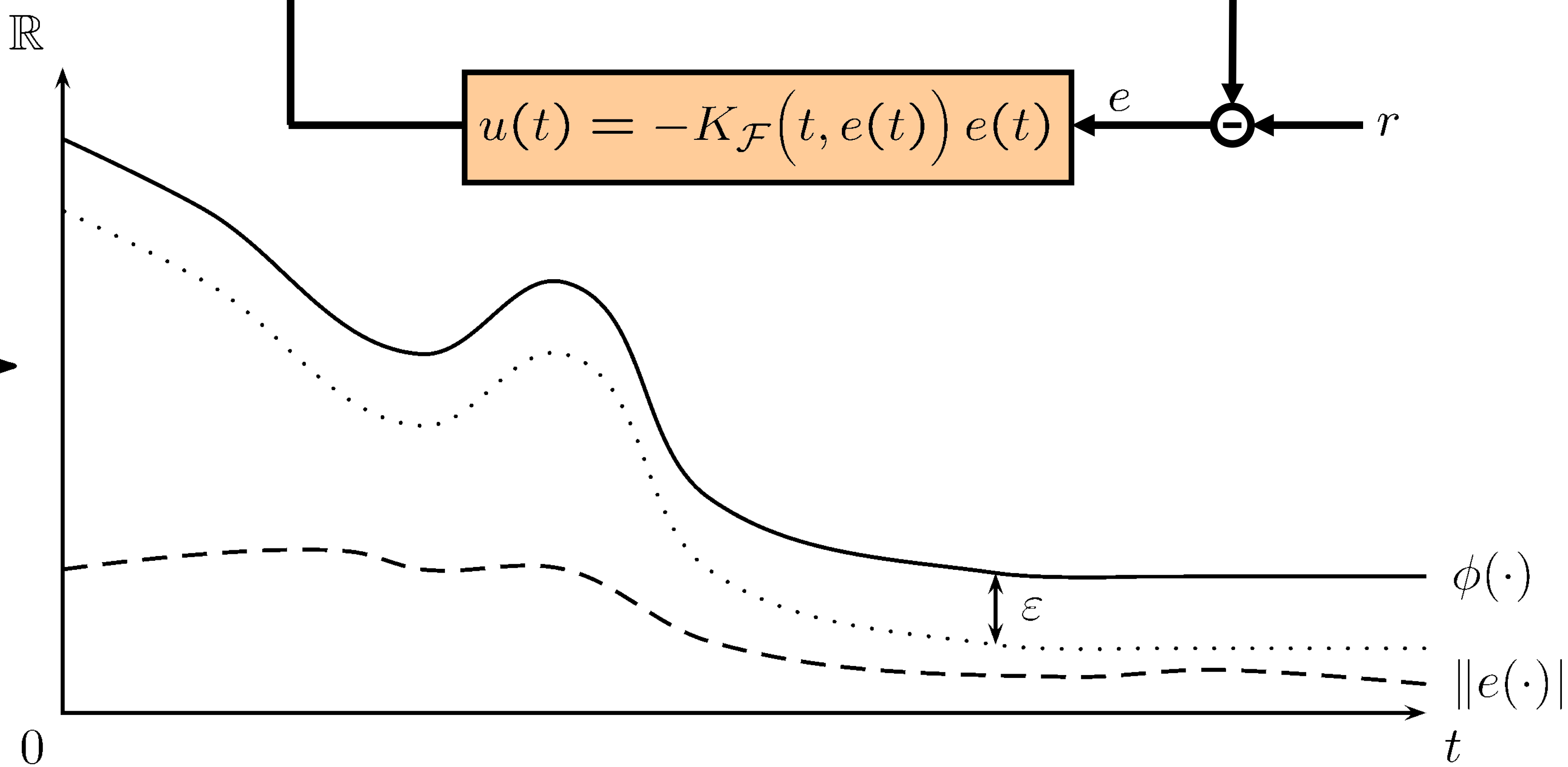
Property 2 $\quad \forall\, \varepsilon > 0\ \forall\, \delta > 0\ \exists\, K > 0\ \forall\, (t,e) \in \mathcal{F}$:

$$\Big[\operatorname{dist}\big(e, \partial\mathcal{F}_t\big) \ge \varepsilon \ \text{ and } \ t \ge \delta \Big] \quad \Rightarrow \quad K_{\mathcal{F}}(t,e) \le K$$

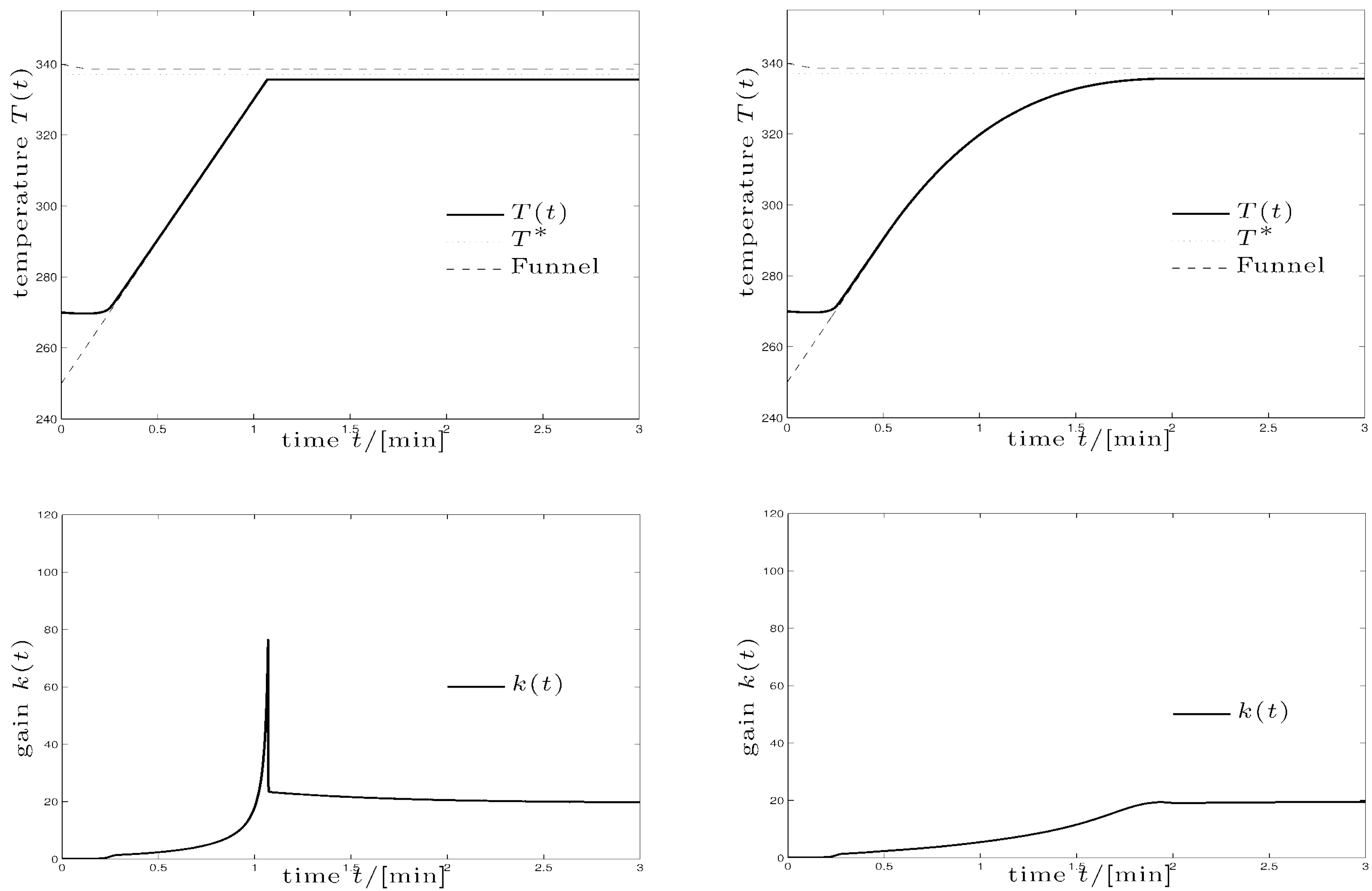
Theorem



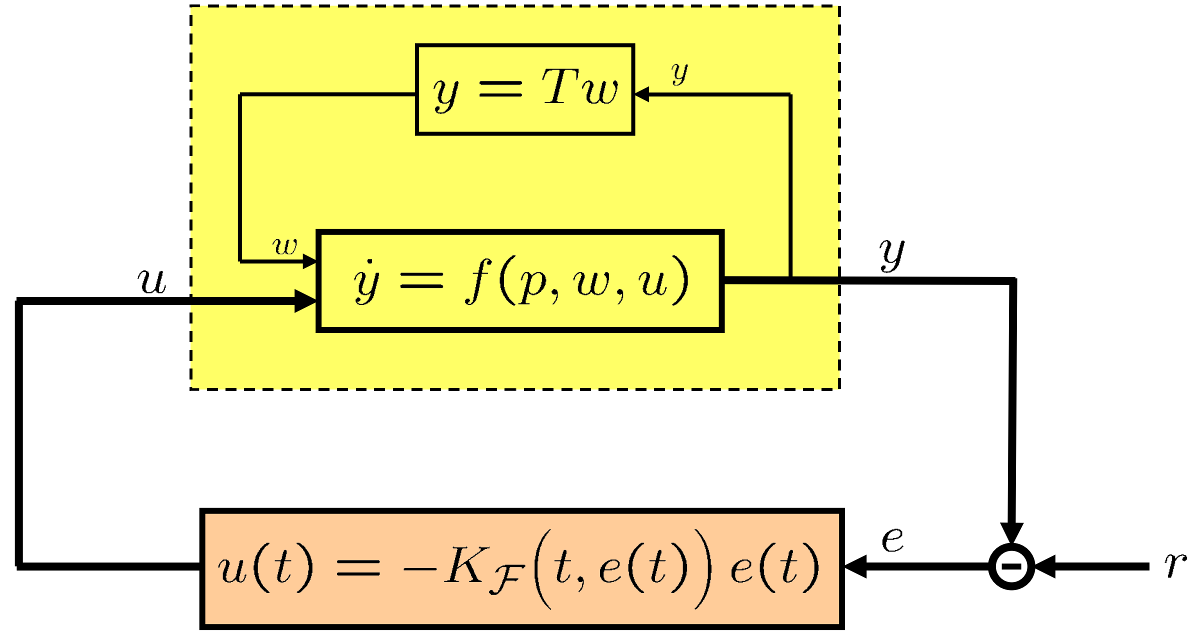
$\Rightarrow$



Applications: Bioreactor



Theorem



$\Rightarrow$