Analogue Implementation of the Funnel Controller

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Content

Scope of funnel control

Aim

Tracking of a reference signal.

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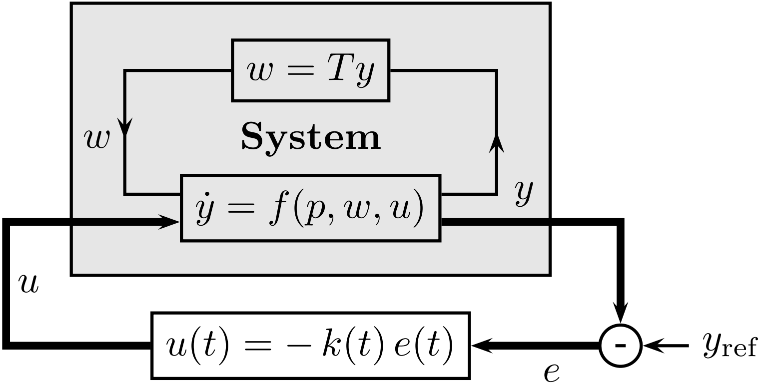
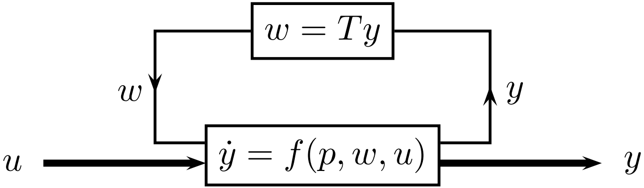


Properties of the system class



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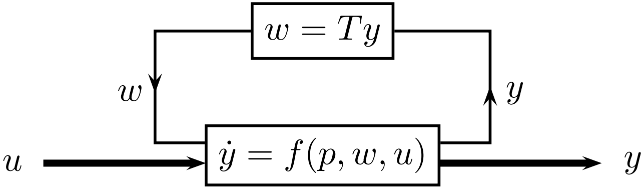

- nonlinear functional differential equations

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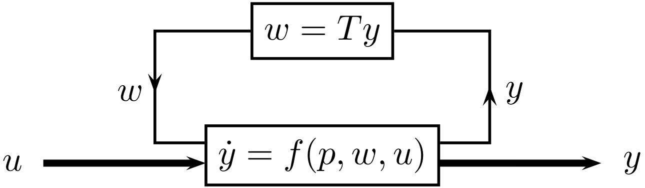

- nonlinear functional differential equations
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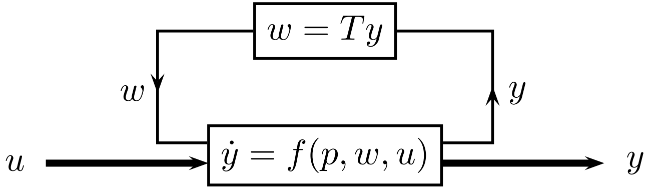

- nonlinear functional differential equations
- bounded-input, bounded-output functional operators such as hysterises and delays
- relative degree one and generalized high-frequency gain property

Control objectives

- **Practical asymptotic stability** of the error, i.e. for a given $\lambda > 0$
$$\exists\, T > 0 \quad \forall\, t \geq T : \quad \big|e(t)\big| < \lambda.$$
- **Prescribed transient behaviour**, e.g. guaranteing an upper bound for the overshoot or an prescribed transient time.
- **Independence of system parameters**, i.e. the same controller works for all systems of the systems class.

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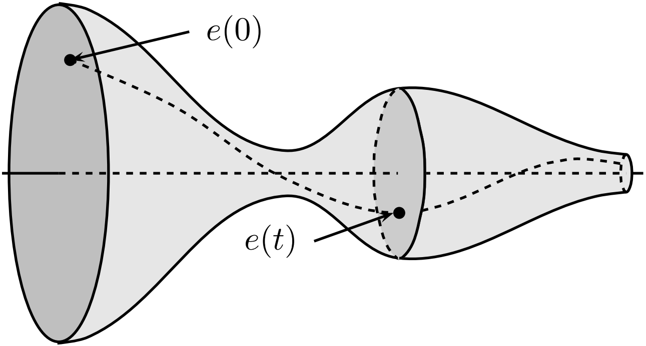
Control objectives $\Leftrightarrow$ prescribed funnel

The funnel $\mathcal{F} \subseteq \mathbb{R}_{\geq 0} \times \mathbb{R}^n$:



Architecture of the funnel controller

The control law:

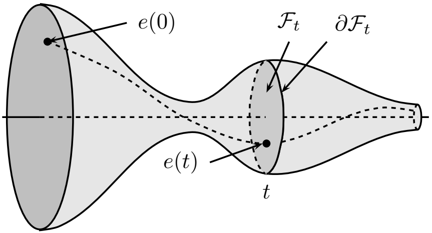
$$u(t) = -k(t)\, e(t)$$

The gain function

$$k(t) = K_{\mathcal{F}}\big(t, e(t)\big)$$

$$K_{\mathcal{F}} : \mathcal{F} \to \mathbb{R}_{\geq 0}$$

Necessary properties of the gain function



Property A

$\forall\, K > 0 \;\; \exists\, \varepsilon > 0 \;\; \forall\, (t, e) \in \mathcal{F}$:

$$\operatorname{dist}(e, \partial\mathcal{F}_t) \leq \varepsilon \quad \Rightarrow \quad K_{\mathcal{F}}(t, e) \geq K$$

Property B

$\forall\, \varepsilon > 0 \; \forall\, \delta > 0 \; \exists\, K > 0 \;\; \forall\, (t, e) \in \mathcal{F}$:

$$\operatorname{dist}(e, \partial\mathcal{F}_t) \geq \varepsilon \quad \text{and} \quad t \geq \delta \quad \Rightarrow \quad K_{\mathcal{F}}(t, e) \leq K$$

Theoretical results

Theorem

For any reference signal any consistent initial data, there **exists a solution** *of the closed-loop initial-value problem.*
Every solution can be extended to a maximal solution $y : [-h, \omega) \to \mathbb{R}^n$ *and every maximal solution has the following properties*

1. $\omega = \infty$,
2. $t \mapsto k(t) = K_{\mathcal{F}}\big(t, e(t)\big)$ **is bounded on** $\mathbb{R}_{\geq 0}$,
3. $\exists \varepsilon > 0 \ \forall t \in \mathbb{R}_{\geq 0}$: $\quad \operatorname{dist}\big(e(t), \partial F(t)\big) \geq \varepsilon$.

Proof in: Ilchmann, Ryan, Trenn (2005): *Tracking control: performance funnels and prescribed transient behaviour*

Further results

- **First funnel controller**
  Ilchmann, Ryan, Sangwin (2002): *Tracking with prescribed transient behaviour*
- **Application to a model of chemical reactors**
  Ilchmann, Trenn (2004): *Input constrained funnel control with applications to chemical reactor models*
- **Higher relative degree systems**
  Ilchmann, Ryan, Townsend (2006): *Tracking with prescribed transient behaviour for nonlinear systems of known relative degree*

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Analogue Implementation

Now to Nagendra ...